Bipolar Vague Relations and their Application to Real-World Problems

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Abstract—**This paper presents an in-depth exploration of bipolar vague relations (BPVRs) and their fundamental properties. We delve into the theoretical underpinnings of BPVRs and demonstrate their application to real-world problems, highlighting their potential to enhance decision-making processes. A novel approach to composite relations in decision-making is introduced, showcasing how the concept of bipolar vague composite relations can be effectively utilized in practical scenarios.**

To illustrate the practical utility of BPVRs, we provide a comprehensive numerical example that guides students in making informed career choices based on their performance across various skill assessments. This example underscores the dual capability of BPVRs to independently evaluate the intervals of satisfaction and dissatisfaction for each option, offering a balanced perspective that is crucial for informed decision-making.

Our findings reveal the significant potential of bipolar vague relations in refining and improving decision-making processes, particularly in contexts characterized by uncertainty and complexity. The application of BPVRs can lead to more accurate, nuanced, and reliable solutions, making them a valuable tool in various domains.

Index Terms—**bipolar fuzzy set, bipolar vague relation, bipolar vague composite relation, decision-making problem.**

I. Introduction and Preliminaries

In fuzzy set (FS) theory, Zadeh [34] defines an FS μ as a class of objects $\digamma$ associated with a membership function (M_{Sh}). This membership function $\mu(\hbar), \hbar \in \digamma$, assigns each object a grade of membership (M_{Sh}) ranging from 0 to 1. Gau and Buehrer [15] introduced the concept of vague sets, which further expands on FSs. According to Gau and Buehrer [15], a vague set κ within a universe $\digamma$ is defined by a pair of functions (t_κ, f_κ), where t_κ and f_κ map elements of $\digamma$ to values between 0 and 1, such that $t_\kappa(\hbar) + f_\kappa(\hbar) \leq 1$ for all $\hbar \in \digamma$. Here, t_κ is the membership function, indicating the degree to which an element $\hbar$ belongs to κ, while f_κ is the non-membership function, representing the degree to which $\hbar$ does not belong to κ. These concepts find applications across various fields, including decision-making, fuzzy control systems, knowledge discovery, and fault diagnosis. Consequently, the theory of vague sets serves as a generalization of the theory of FSs.

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In the early days of vague algebra, Biswas created vague groups (VGs), vague normal groups (VNGs), vague homomorphisms, and vague relations (VRs). These are all versions of fuzzy groups (FGs), fuzzy normal groups (FNGs), fuzzy homomorphisms, and fuzzy relations (FRs). In the next part of his research, Ramakrishna [33], [31], [27], [30], [28], [29], [32], [26] looked at how to describe cyclic groups using vague groups (VGs), vague normal groups (VNGs), homologous vague groups, vague normalizers, vague centralizers, vague weights, and vague graphs. Eswarlal [9], [7], [6], [8] advanced the study of L-vague sets, L-vague relations, and L-vague groups, where L represents a complete lattice that adheres to the infinite meet distributive law. His work also delved into vague ideals in semirings, normal vague ideals in semirings, Boolean vague sets, Boolean vague prime ideals, and Boolean vague maximal ideals in general rings. Human decisions often rely on dual or bipolar judgmental thinking, encompassing both positive (+ve) and negative (-ve) aspects. Recognizing this, Lee [20] introduced the concept of bipolar fuzzy sets (BFSs), which account for this duality in human decision-making processes.

A BFS is defined as a pair (μ^+, μ^-), where $\mu^+ : \digamma \to [0,1]$ and $\mu^- : \digamma \to [-1,0]$ are mappings. BFSs extend the concept of FSs by having a membership degree range of $[-1,1]$. In a BFS, a membership degree of 0 indicates that the element is irrelevant to the property in question. A membership degree in the range $(0,1]$ signifies that the element somewhat satisfies the property, while a membership degree in the range $[-1,0)$ indicates that the element somewhat satisfies the counter-property. Consider a universal set $\digamma$ and a set κ defined over $\digamma$ by a positive membership function μ_κ^+ and a negative membership function μ_κ^-. Specifically, $\mu_\kappa^+ : \digamma \to [0,1]$ and $\mu_\kappa^- : \digamma \to [-1,0]$. Then, κ is termed a BFS over $\digamma$ and can be expressed as $\kappa = \, < \hbar, \mu_\kappa^+(\hbar), \mu_\kappa^-(\hbar) > \mid \hbar \in \digamma$.

Flora et al. [10] extended the study of bipolarity to vague sets and studied bipolar vague subgroups and bipolar vague normal subgroups. Let $\digamma$ be a universe of discourse, and κ be an object over $\digamma$. Then κ is known as a bipolar vague set, which is of the form: $\kappa = \{< \hbar, [t_\kappa^+(\hbar), 1 - f_\kappa^+(\hbar)], [-1 - f_\kappa^-(\hbar), t_\kappa^-(\hbar)] > \mid \hbar \in \digamma\}$, where $[t_\kappa^+, 1 - f_\kappa^+] : \digamma \to [0,1]$ and $[-1 - f_\kappa^-, t_\kappa^-] : \digamma \to [-1,0]$ are mappings so that $t_\kappa^+ + f_\kappa^+ \leq 1$ and $-1 \leq t_\kappa^- + f_\kappa^-$. The positive (+ve) membership interval $[t_\kappa^+(\hbar), 1 - f_\kappa^+(\hbar)]$ symbolizes the interval of satisfaction (belongingness) of an element $\hbar$ to the property corresponding to a BFS κ, and the negative (-ve) membership interval $[-1 - f_\kappa^-(\hbar), t_\kappa^-(\hbar)]$ symbolizes the interval of satisfaction (belongingness) of $\hbar$ to some implicit counter property of κ. In simple terms, we notate $v_\kappa^+ = [t_\kappa^+, 1 - f_\kappa^+]$ and $v_\kappa^- = [-1 - f_\kappa^-, t_\kappa^-]$ are used to notate a bipolar vague set.

Chakraborty and Das [5], [3], [4] studied fuzzy relations in 1983. Murali [25] extended the study to fuzzy equivalence relations in 1989. Bustince and Burillo [2] explored the study of structures on IFRs in 1996. Fuzzy equivalence relations (FEqR) and fuzzy functions were explored by Lee [21]. Later, Hur et al. [17] studied interval-valued fuzzy relations in 2009. Khan et al. [18], [19] explored the study of vague relations and vague groups in 2007. Lee and Hur [22] introduced bipolar fuzzy relations in 2019 and established many crucial properties. In 2021, Gaketem and Khamrot [12] gave the concepts of bipolar fuzzy weakly interior ideals of semigroups and some interesting properties of bipolar fuzzy weakly interior ideals of semigroups. In 2022, Luo and Gao [23] introduced the degree of independence in a bipolar fuzzy graph setting. The work of Gaketem et al. [14] introduced the idea of bipolar fuzzy comparative UP-filters in UP-algebras and looked into their basic properties. Lu et al. [24] came up with the idea of a bipolar fuzzy influence graph in 2023. This type of graph uses positive and negative membership functions to show uncertainties that are positive and negative, respectively. Additionally, Gaketem et al. [11] talked about the ideas of cubic bipolar fuzzy subsemigroups, describing their properties and looking at how they relate to other subsemigroups and their own features. Gong and Gao [16] introduced a bipolar fuzzy topological graph to assess the features of bipolar fuzzy systems. In 2024, Gaketem and Prommai [13] introduced the concept of bipolar fuzzy bi-interior ideals and explored several of their properties. In BFSs, we deal with single positive and negative membership values. In contrast, in bipolar vague sets, we can deal with positive intervals (for satisfaction) and negative intervals (for dissatisfaction). So, keeping this advantage in mind, we explored and studied bipolar vague relations in this paper.

Now, we recall definitions that are related to this section.

Definition I.1 [5] A FR on $\digamma$ and Y is a fuzzy subset μ of the Cartesian product $\digamma \times Y$. A FR on a set $\digamma$ is a fuzzy subset μ of the Cartesian product $\digamma \times \digamma$.

Definition I.2 [19] A vague relation (VR) on $\digamma$ and Y is a vague subset of the Cartesian product of $\digamma \times Y$. A vague relation on a set $\digamma$ is a vague subset of the Cartesian product $\digamma \times \digamma$.

Definition I.3 [22] $B_R = (B_R^P, B_R^N)$ is known as BFR from the universe of discourse $\digamma$ to the universe of discourse Y, if $B_R^P : \digamma \times Y \to [0,1]$, and $B_R^N : \digamma \times Y \to [-1,0]$ are mappings, i.e., $B_R \in BPF(\digamma \times Y)$.

A BPFR from $\digamma$ to $\digamma$ is called a BPFR on $\digamma$.

II. Bipolar vague relations

This section introduces the concept of bipolar vague relations (BPVRs) and explores their fundamental properties. Additionally, we demonstrate the application of BPVR theory to real-world decision-making problems.

Bipolar vague relations offer significant advantages in various fields:

1) Enhanced Decision-Making: BPVRs provide a more nuanced approach to decision-making by considering both positive and negative aspects simultaneously, leading to more balanced and informed choices.
2) Improved Data Analysis: By accommodating vague and uncertain information, BPVRs enable more accurate and flexible analysis of complex data sets.
3) Robust Modeling: BPVRs facilitate the creation of models that can handle imprecise and bipolar information, improving the robustness and reliability of these models in practical applications.
4) Versatile Applications: The theory of BPVRs can be applied across diverse domains such as economics, healthcare, engineering, and social sciences, enhancing problem-solving capabilities in these areas.

We will now extend the concept of relations to include bipolar vague sets (BVS).

Definition II.1 Let $K = (\digamma, V_K^P, V_K^N)$ be a bipolar vague set on $\digamma$ and $B = (Y, V_B^P, V_B^N)$ be a bipolar vague set on Y. Then we define a bipolar vague relation (BPVR) $R1$ as a bipolar vague subset of the Cartesian product $K \times B = R \subset \digamma \times Y$ such that $R1 = \{< (\hbar, s), V_{R1}^P(\hbar, s), V_{R1}^N(\hbar, s) >| \hbar \in K, s \in B\}$, where $V_{R1}^P : K \times B \to [0,1]$, $V_{R1}^N : K \times B \to [-1,0]$ and $V_{R1}^P(\hbar, s) = V_{K\times B}^P(\hbar, s) = \text{rmin}\{V_K^P(\hbar), V_B^P(s)\}, V_{R1}^N(\hbar, s) = V_{K\times B}^N(\hbar, s) = \text{rmax}\{V_K^N(\hbar), V_B^N(s)\}$ for all $\hbar \in \digamma, s \in Y$.

In particular, a BPVR form K to K is called a BPVR on K.

Definition II.2 The null BPVR (resp., the whole BPVR) on K, denoted by $R_0 =< V_{R_0}^P, V_{R_0}^N >$ (resp., $R_I =< V_{R_I}^P, V_{R_I}^N >$) is defined as follows: for each $(\hbar, s) \in K \times B$, $V_{R_0}^P(\hbar, s) = [0,0] = V_{R_0}^N(\hbar, s)$ (resp., $V_{R_I}^P(\hbar, s) = [1,1]$ and $V_{R_I}^N(\hbar, s) = [-1,-1]$).

Definition II.3 Let R be a BPVR from K to B. Then the inverse of R denoted by $R^{-1} =< (V_R^P)^{-1}, (V_R^N)^{-1} >$ is a BPVR from B to K defined as follows: for each $(b_r, a_r) \in B \times K$, $R^{-1}(b_r, a_r) = R(a_r, b_r)$, i.e., $(V_R^P)^{-1}(b_r, a_r) = (V_R^P)(a_r, b_r)$ and $(V_R^N)^{-1}(b_r, a_r) = (V_R^N)(a_r, b_r)$.

Definition II.4 Let $R1$ be a BPVR from K to B. Then the complement of $R1$ denoted by $R1^c =< (V_{R1}^P)^c, (V_{R1}^N)^c >$ is a BPVR from B to K defined as follows: for each $(a_r, b_r) \in B \times K$, $(V_{R1}^P)^c(a_r, b_r) = 1 - (V_{R1}^P)(a_r, b_r)$ and $(V_{R1}^N)^c(a_r, b_r) = -1 - (V_{R1}^N)(a_r, b_r)$.

Definition II.5 Let $R_1 =< (V_{R_1}^P), (V_{R_1}^N) >$ and $R_2 =< (V_{R_2}^P), (V_{R_2}^N) >$ be two bipolar vague relations from K to B. Then their union is given by $R_1 \cup R_2 =< (V_{R_1\cup R_2}^P), (V_{R_1\cup R_2}^N) >$ such that

$$V_{R_1\cup R_2}^P(\hbar, s) = \text{rmax}\{V_{R_1}^P(\hbar, s), V_{R_2}^P(\hbar, s)\} \text{ and }$$
$$V_{R_1\cup R_2}^N(\hbar, s) = \text{rmin}\{V_{R_1}^N(\hbar, s), V_{R_2}^N(\hbar, s)\}.$$

Definition II.6 Let R_1 and R_2 be two bipolar vague relations from K to B. Then their intersection is given by $R_1 \cap R_2 =< (V_{R_1\cap R_2}^P), (V_{R_1\cap R_2}^N) >$ such that

$$V_{R_1\cap R_2}^P(\hbar, s) = \text{rmin}\{V_{R_1}^P(\hbar, s), V_{R_2}^P(\hbar, s)\} \text{ and }$$
$$V_{R_1\cap R_2}^N(\hbar, s) = \text{rmax}\{V_{R_1}^N(\hbar, s), V_{R_2}^N(\hbar, s)\}.$$

Definition II.7 (Composition of two BPVRs) Let $R_1 \in BPVR(K, B)$ and $R_2 \in BPVR(B, C)$, we define the composite relation $S = R_1 \circ R_2 =< V_{R_1\circ R_2}^P, V_{R_1\circ R_2}^N >$ is a BPVR of K to C defined by

$$V_{R_1\circ R_2}^P(\hbar, z) = \max_{s\in Y}\{\text{rmin}\{V_{R_1}^P(\hbar, s), V_{R_2}^P(s, z)\}\}$$
$$V_{R_1\circ R_2}^N(\hbar, z) = \min_{s\in Y}\{\text{rmax}\{V_{R_1}^N(\hbar, s), V_{R_2}^N(s, z)\}\}.$$

Definition II.8 (Composition of a BVS and a BPVR) Let K be a BVS of the universe $\digamma$ and $R \in BPVR(\digamma, Y)$, we define the composition of R with K, denoted by $B = R \circ K =< V_{R\circ K}^P, V_{R\circ K}^N >$ is a BVS in Y defined by

$$V^P_{R\circ K}(s) = \max_{\hbar\in F}\{\text{rmin}\{V^P_K(\hbar), V^P_R(\hbar, s)\}\}$$
$$V^N_{R\circ K}(s) = \min_{\hbar\in F}\{\text{rmax}\{V^N_K(\hbar), V^N_R(\hbar, s)\}\}.$$

Definition II.9 Let $R1$ be a BPVR on F. Then $R1$ is said to be reflexive, if for each $\hbar \in F$ $V^P_{R1}(\hbar,\hbar) = [1,1]$,$V^N_{R1}(\hbar,\hbar) = [-1,-1]$, anti-reflexive if for each $\hbar \in F$, $V^P_{R1}(\hbar,\hbar) = [0,0]$ and $V^N_{R1}(\hbar,\hbar) = [0,0]$.

Definition II.10 Let $R1$ be a BPVR on F. Then $R1$ is said to be symmetric, if for each $\hbar, s \in F$, $V^P_{R1}(\hbar,s) = V^P_{R1}(s,\hbar)$ and $V^N_{R1}(\hbar,s) = V^N_{R1}(s,\hbar)$, anti-symmetric if for each $\hbar, s \in F$, $\hbar \neq s$, $V^P_{R1}(\hbar,s) \neq V^P_{R1}(s,\hbar)$ and $V^N_{R1}(\hbar,s) \neq V^N_{R1}(s,\hbar)$.

Definition II.11 Let $R1$ be a BPVR on F. Then $R1$ is said to be transitive: $R1^2 := R1 \circ R1 \subseteq R1$, if for each $\hbar, s \in F$, $V^P_{R1}(\hbar,s) = V^P_{R1}(s,\hbar)$ and $V^N_{R1}(\hbar,s) = V^N_{R1}(s,\hbar)$, anti-symmetric if for each $\hbar, s \in F$, $\hbar \neq s$, $V^P_{R1}(\hbar,s) \neq V^P_{R1}(s,\hbar)$ and $V^N_{R1}(\hbar,s) \neq V^N_{R1}(s,\hbar)$.

Definition II.12 Let $R1$ be a BPVR on F. Then $R1$ is said to be an equivalence relation if $R1$ is reflexive, symmetric, and transitive.

Now, in the following section, we will give an application for BPVR.

III. Application of Bipolar Vague Relation

Let $S = \{s_1, s_2, s_3, ..., s_n\}$ be a set of students. Let A be a BVS of the universe S, and R be a BPVR of the universe ES with the universe J. Then the composition $B = AoR$ demonstrates the choice of the student in terms of the job as a BVS of J, with bipolar vague value defined for all $j \in J$ by

$$V^P_B(j) = [\text{rmax}_{e\in ES}\{\text{rmin}\{V^P_\kappa(j), V^P_{R1}(s,j)\}\}] \quad (1)$$
$$V^N_B(j) = [\text{rmin}_{e\in ES}\{\text{rmax}\{V^N_\kappa(j), V^N_{R1}(s,j)\}\}]. \quad (2)$$

If the employability skills of a given student "s" are examined and described in terms of a BVS A of the universe ES, then the student "s" is assumed to suggest a job in terms of BVS B of the universe J through a BPVR R of the "Strategy-Knowledge" from ES to J, which is presumed to be given by a talent acquisition manager who can elaborately translate his perception of the vagueness that is involved in a bipolar vague-valued degree of association between employability skills and jobs.

Now let us consider students $S_i \in S$, R is a BPVR from ES to J, and define a BPVR Q from S to J. Then (1) and (2) become

$$V^P_T(s,j) = [\text{rmax}_{e\in ES}\{\text{rmin}\{V^P_Q(s,e), V^P_R(e,j)\}\}]$$
$$V^N_T(s,j) = [\text{rmin}_{e\in ES}\{\text{rmax}\{V^N_Q(s,e), V^N_R(e,j)\}\}],$$

where $T = R\circ Q$. By R and Q, we can compute and solve T. From the knowledge of Q and T, one may compute, when it exists, the R for which V^P_T is greatest, such that $T = R \circ Q$, the most appropriate and significant bipolar vague relation that translates the higher degrees of associations of suitable jobs, is an approach to "Strategy-Knowledge".

Numerical Example: Providing relevant information to students for making informed career choices cannot be overstated. Effective career guidance is crucial, as students' numerous challenges due to its absence significantly impact their job choices and performance. Therefore, it is essential for students to receive proper knowledge about job determination or selection to enhance their planning, preparation, and performance. Making decisions about educational matters is increasingly common today, with students often seeking the services of counsellors and advisors to receive the best guidance and make optimal choices.

In our study, we assessed the employability skills of ten engineering students, examining their technical, analytical, presentation, communication, and organizational abilities through a series of relevant tests and presentations. We employed a "strategy-knowledge" approach, commonly used by talent acquisition managers, to analyze the correlation between these skills and job suitability. This analysis utilized descriptive linguistic terms—ranging from "extremely good" to "worst," as detailed in Table I.

Table II provides a comprehensive overview of each student's employability skills based on the test results. Furthermore, Table III presents an evaluation of each student's capability to secure various job roles, derived from the computed composite relations. The insights gained from this study offer targeted recommendations for students on which skills to focus on for enhancing their career prospects and achieving their desired positions.

From Table III, we can comprehensively evaluate the capabilities of each student in relation to different job roles. By analyzing the data, we can offer tailored career guidance to each student, ensuring that their strengths and preferences are matched with suitable job opportunities. The job with the highest V^P_T value is identified as the most fitting position for the student, indicating where they are likely to excel and find satisfaction.

IV. Conclusion

One of the primary advantages of the Bipolar Vague Relation (BPVR) in decision-making processes is its ability to independently determine the intervals of satisfaction and dissatisfaction for each option under evaluation. This unique feature enables decision-makers to make more informed and balanced choices, as they can simultaneously consider both the positive and negative aspects of each alternative. This dual consideration helps in identifying the most suitable options based on a comprehensive evaluation of their strengths and weaknesses.

The significance of this work lies in the application of BPVRs to model and solve real-world problems, particularly in scenarios where uncertainty and vagueness are inherent. By utilizing BPVRs, we can more accurately capture the complexities and nuances of real-world decision-making, leading to solutions that are robust, reliable, and better aligned with the intricate nature of practical challenges.

TABLE I: $R : ES \rightarrow J$

$R : ES \rightarrow J$	Core Job	BPO	Teaching Job	Management Job	Self Employment
Technical Skills	[0.08,0.09] [-0.02,-0.01]	[0.02,0.09] [-0.06,-0.01]	[0.08,0.09] [-0.02,-0.01]	[0.01,0.07] [-0.02,-0.03]	[0.07,0.08] [-0.01,-0.02]
Analytical Skills	[0.08,0.05] [-0.01,-0.02]	[0.05,0.06] [-0.03,-0.04]	[0.06,0.09] [-0.01,-0.03]	[0.08,0.09] [-0.03,-0.02]	[0.07,0.08] [-0.04,-0.03]
Presentation Skills	[0.08,0.07] [-0.04,-0.02]	[0.08,0.01] [0.06,-0.02]	[0.00,0.06] [-0.03,-0.04]	[0.02,0.07] [-0.03,-0.04]	[0.00,0.05] [-0.02,-0.07]
Communication Skills	[0.05,0.03] [-0.01,-0.05]	[0.08,0.00] [-0.06,-0.02]	[0.07,0.06] [-0.03,-0.01]	[0.07,0.07] [-0.05,-0.01]	[0.09,0.00] [-0.04,-0.01]
Organisation Skills	[0.01,0.02] [-0.03,-0.04]	[0.01,0.08] [-0.02,-0.04]	[0.05,0.07] [-0.03,-0.04]	[0.09,0.07] [-0.02,-0.01]	[0.08,0.01] [-0.01,-0.02]

TABLE II: $Q : S \rightarrow ES$

$Q : S \rightarrow ES$	Technical Skills	Analytical Skills	Presentation Skills	Communication Skills	Organisation Skills
S_1	[0.08,0.08] [-0.03,-0.02]	[0.06,0.01] [-0.02,-0.01]	[0.02,0.08] [-0.06,-0.02]	[0.06,0.01] [-0.01,-0.07]	[0.01,0.06] [-0.01,-0.02]
S_2	[0.00,0.08] [-0.02,-0.03]	[0.04,0.04] [-0.03,-0.02]	[0.06,0.01] [-0.04,-0.04]	[0.01,0.07] [-0.01,-0.06]	[0.01,0.08] [-0.02,-0.01]
S_3	[0.08,0.07] [-0.04,-0.02]	[0.08,0.01] [0.06,-0.02]	[0.00,0.06] [-0.03,-0.04]	[0.02,0.07] [-0.03,-0.04]	[0.00,0.05] [-0.02,-0.07]
S_4	[0.04,0.01] [-0.02,-0.01]	[0.05,0.04] [-0.02,-0.08]	[0.03,0.04] [-0.01,-0.06]	[0.01,0.02] [-0.02,-0.05]	[0.03,0.04] [-0.02,-0.01]
S_5	[0.08,0.08] [-0.05,-0.05]	[0.05,0.04] [-0.06,-0.02]	[0.02,0.08] [-0.04,-0.02]	[0.03,0.07] [-0.03,-0.01]	[0.08,0.07] [-0.03,-0.01]
S_6	[0.05,0.05] [-0.05,-0.04]	[0.08,0.03] [-0.06,-0.02]	[0.02,0.05] [-0.04,-0.02]	[0.04,0.06] [-0.04,-0.02]	[0.05,0.05] [-0.04,-0.02]
S_7	[0.04,0.02] [-0.06,-0.05]	[0.06,0.04] [-0.06,-0.01]	[0.08,0.09] [-0.09,-0.01]	[0.08,0.09] [-0.07,-0.01]	[0.04,0.05] [-0.07,-0.01]
S_8	[0.05,0.07] [-0.04,-0.03]	[0.04,0.04] [-0.05,-0.01]	[0.04,0.05] [-0.04,-0.01]	[0.04,0.06] [-0.03,-0.02]	[0.06,0.08] [-0.03,-0.02]
S_9	[0.09,0.07] [-0.06,-0.03]	[0.06,0.04] [-0.07,-0.02]	[0.05,0.05] [-0.03,-0.02]	[0.04,0.05] [-0.02,-0.01]	[0.09,0.07] [-0.02,-0.01]
S_{10}	[0.02,0.01] [-0.05,-0.04]	[0.07,0.08] [-0.06,-0.02]	[0.08,0.05] [-0.04,-0.03]	[0.07,0.04] [-0.03,-0.02]	[0.02,0.03] [-0.09,-0.02]

TABLE III: $T = R \circ Q$

$T = R \circ Q$	Core Job	BPO	Teaching Job	Management Job	Self Employment
S_1	[0.08,0.08] [-0.02,-0.05]	[0.06,0.08] [-0.03,-0.02]	[0.08,0.08] [-0.06,-0.02]	[0.06,0.07] [-0.06,-0.02]	[0.07,0.08] [-0.02,-0.02]
S_2	[0.05,0.08] [-0.02,-0.05]	[0.06,0.08] [-0.03,-0.04]	[0.06,0.08] [-0.04,-0.02]	[0.06,0.07] [-0.04,-0.03]	[0.06,0.08] [-0.03,-0.04]
S_3	[0.08,0.07] [-0.02,-0.04]	[0.05,0.07] [-0.04,-0.04]	[0.08,0.07] [-0.03,-0.04]	[0.08,0.07] [-0.03,-0.03]	[0.07,0.07] [-0.04,-0.04]
S_4	[0.05,0.04] [-0.02,-0.05]	[0.05,0.04] [-0.02,-0.06]	[0.05,0.04] [-0.02,-0.03]	[0.05,0.04] [-0.02,-0.03]	[0.05,0.04] [-0.02,-0.04]
S_5	[0.08,0.08] [-0.03,-0.02]	[0.05,0.08] [-0.05,-0.02]	[0.08,0.08] [-0.04,-0.02]	[0.08,0.07] [-0.04,-0.03]	[0.08,0.08] [-0.04,-0.02]
S_6	[0.08,0.05] [-0.03,-0.02]	[0.05,0.05] [-0.05,-0.02]	[0.06,0.06] [-0.04,-0.02]	[0.08,0.06] [-0.04,-0.03]	[0.07,0.05] [-0.04,-0.02]
S_7	[0.06,0.07] [-0.03,-0.01]	[0.08,0.07] [-0.06,-0.01]	[0.08,0.06] [-0.08,-0.01]	[0.08,0.07] [-0.06,-0.03]	[0.08,0.08] [-0.04,-0.02]
S_8	[0.05,0.07] [-0.03,-0.02]	[0.04,0.08] [-0.04,-0.02]	[0.05,0.07] [-0.04,-0.02]	[0.06,0.07] [-0.04,-0.03]	[0.06,0.07] [-0.04,-0.02]
S_9	[0.08,0.07] [-0.02,-0.02]	[0.05,0.07] [-0.06,-0.02]	[0.08,0.07] [-0.03,-0.02]	[0.09,0.07] [-0.03,-0.03]	[0.08,0.07] [-0.04,-0.02]
S_{10}	[0.07,0.05] [-0.03,-0.03]	[0.08,0.06] [-0.05,-0.03]	[0.08,0.08] [-0.04,-0.02]	[0.08,0.08] [-0.04,-0.03]	[0.07,0.08] [-0.04,-0.03]

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