Green Networking Approach Using Conic Optimization

Bimal Chandra Das, *Member, IAENG,* Shahina Haque, *Member, IAENG,* Nusrat Jahan, *Member, IAENG,* and Momotaz Begum, *Member, IAENG*

***Abstract*—This study introduces a mixed integer second-order cone programming (MISOCP) model called the green Ellipsoid model for power saving in communication networks. The goal of this research is to optimize power consumption in communication networks by applying the conic optimization technique which is an emerging branch of research. The consumed power in communication networks is minimized considering network data variation using robust optimization. We apply the ellipsoidal uncertainty set to grant a variation in the network data and presented the model in the form of MISOCP. To scale down the total energy utilization in the network, the study proposes shutting down some unnecessary links throughout the entire network. By identifying and deactivating these redundant or unused links, the overall power consumption is reduced. Based on the numerical results obtained, the proposed green Ellipsoid model demonstrates the ability to save consumed power in comparison to the existing green Hose approach. The results indicate that the green Ellipsoid model exceeds the green Hose approach in terms of power savings. This suggests that the introduced model is more adequate in optimizing power utilization and achieving energy efficiency in the communication networks. An additional advantage of the green Ellipsoid model is its ability to accommodate data variation across the network through a single parameter. Acknowledging that MISOCP is a known NP-hard problem, the study also proposes an improved method to minimize computation time. This is achieved by leveraging optimization software tools that make the model tractable. By utilizing these tools, the presented model can be efficiently solved within an acceptable time.**

***Index Terms*—second-order cone, data fluctuation, optimization, green computing, power consumption, ellipsoid.**

I. Introduction

POWER saving in communications networks has a crucial aspect in today's technology-driven world. With the increasing demand for data transmission and communication services, optimizing power consumption has become essential to ensure sustainable and efficient network operations. By implementing various strategies and techniques, such as shutting down unused links or employing robust optimization models significant power savings can be achieved [1]. This not only reduces energy costs but also contributes to environmental sustainability [2].

The world's temperature is rising up day by day due to global warming and it is a concerning issue nowadays. The computers and electronics industry contributes significantly to energy consumption and the emission of greenhouse gases. However, by implementing green computing practices, it is desirable to minimize energy utilization and mitigate the over-radiation of greenhouse gases [3]. Green computing focuses on adopting energy-efficient automation and practices to minimize the environmental shock of the industry. By embracing green computing, we can work towards reducing the energy consumption associated with computers and electronics, thereby contributing to the efforts in combating global warming [4], [5]. In this context, this kind of research explores the importance of power savings in communications networks and highlights the potential benefits of implementing effective power-saving measures.

There has been a significant amount of research conducted in the field of green computing and effective power savings. Researchers have explored various strategies and technologies to reduce energy consumption in computing systems, such as optimizing hardware components, developing energy-efficient algorithms [4], implementing power management techniques, and promoting virtualization and cloud computing [3]. These efforts aim to minimize the environmental impact of computing activities and improve energy efficiency in order to achieve sustainable and eco-friendly computing practices.

A. Related Works

For minimizing energy utilization, the work done by Bianzino et al. [6] which presented the green Pipe model sounds, interesting and relevant to the goal of reducing energy consumption in networks. By fixing the network traffic demands and formulating a model to reduce energy in communications networks based on these requirements, they likely aimed to optimize energy efficiency in network operations. This type of research is valuable in promoting sustainable and environmentally friendly practices in the field of networking.

Scaling down power consumption by turning off redundant links is a common approach in green computing. By identifying and deactivating unnecessary or underutilized links, energy can be saved without compromising the overall performance of the network. This strategy aligns with the principles of green computing, which prioritize energy efficiency and sustainability. The work of Bianzino et al. in this area, contributes to the development of practical solutions for

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reducing power consumption in networks. Indeed, the green Pipe model's efficiency is enhanced by leveraging previously known traffic information. By utilizing this information, the model can make more accurate predictions and optimizations regarding power consumption in communication networks. This approach allows for better resource allocation and the ability to identify and exploit opportunities for energy savings.

By incorporating prior knowledge of traffic patterns, the green Pipe model can achieve higher efficiency in reducing power consumption, contributing to more sustainable and environmentally friendly network operations. But in reality, network traffic can fluctuate, making it challenging for operators to accurately estimate the actual traffic. This variability in traffic patterns can pose difficulties in implementing power-saving strategies that rely on fixed traffic demands. To address this issue, researchers and network operators have explored dynamic traffic management techniques, such as adaptive routing algorithms and traffic prediction models. These approaches aim to effectively adapt network resources and power usage considering real-time network traffic conditions, allowing for more efficient power management in response to changing traffic patterns.

In scenarios where traffic patterns vary due to factors such as user demand, network congestion, or dynamic workloads, the fixed nature of the green Pipe model may not effectively adapt to these changes. As a result, the model may not provide optimal power-saving solutions in such dynamic environments. In this research, we followed the power model presented by Bianzino et al. [6] and incorporated traffic-variation adaptability in it.

Duffield et al. [7] developed the Hose model, which differs from the Pipe model approach in terms of not requiring exact traffic demands. This suggests that the author's approach may offer an alternative method for reducing power consumption in networks without relying on precise traffic demand information. The Hose model could potentially provide more flexibility and adaptability in optimizing energy efficiency, allowing for more dynamic adjustments in network operations. The Hose model is a traffic management approach that sets limits on the total incoming and outgoing data for every node in a network. By implementing these boundaries, network administrators can ensure efficient and reliable communication within the network.

A model is developed by Das et al. [8] for network power utilization, considers fluctuations in traffic demands using mathematical optimization. For obtaining a solution, the optimization problem is transfigured into an MISOCP problem using conic duality. This optimization model uses fluctuations in data requirements among the source and destination nodes. This paper compares the green HLT approach with the introduced model.

The work presented by Mao et al. [9] focused on creating a reliable and secure way for unmanned aerial vehicles (UAVs) to transmit and compute tasks in mobile edge computing (MEC) networks. It addresses challenges like uncertain channels and formulates an energy-saving problem, optimizing offloading time, CPU frequency, beamforming, and UAV trajectory, solved through a proposed algorithm that outperforms existing methods.

Zhang and Li [10] introduced a model to provide an improved scheduling technique for better resource allocation in cloud computing, using virtual machine migration and enhanced particle swarm optimization. Their proposed method prevents low server or excessive overloads through scheduling tasks to physical servers and optimal allocation. This method aims to boost efficiency, reduce energy consumption, and improve execution time compared to existing approaches.

Ghayoor et al. [11] applied mixed-integer linear programming to introduce a new approach for optimizing a power network's reliability. The model is formulated applying the system average disturbance duration index and average energy not supplied index considering the feeder's collapse rate. They also used robust optimization techniques to defeat the unstable environment and the constraints include acceptable voltage drop, standard ranges for current magnitude and electric power, power balance, and power law. The proposed approach enhances network performance and cost savings by minimizing energy usage and the work is demonstrated through real-world case studies.

B. Robust Optimization

A subclass of optimization problem where information about some parameters is not provided directly rather than given in a predefined set, and if the problem is solved considering the worst case, then it is called robust optimization. It has been established in the area of optimization [12], [13], [14], [15] in the last decades. In this context, the predefined set is known as the uncertainty set. We are dealing with data fluctuation in this research and an uncertainty set is used to allow fluctuations in data demands. There are a lot of research have been conducted in real-life fields using robust optimization such as mechanics [16], communications [17], [18], management [19], finance [20], and control [21], [22]. The readers may cite to Boyd & Vandenberghe [15] and Ben-Tal & Nemirovski [12] for more studies on robust optimization.

C. Problem Statement

Although considerable work has been done to minimize power consumption considering data variations, our aim is to develop a robust optimization approach using an ellipsoidal uncertainty set that can allow data fluctuations over the network. To our knowledge, no previous work has been done solely using ellipsoidal uncertainty to minimize network power utilization.

The work developed by Bianzino et al. [6] proposed the Pipe model approach, which is described considering the fixed data requirements. The Pipe model is not robust to data variations. The green Hose model proposed by Duffield et al. [7] is robust, but the achievement in energy minimization is smaller than the Pipe model approach. Our problem is to propose an alternate model using conic optimization that can deal with data fluctuations across the backbone network and also to claim the performances of the green Hose approach.

D. Contributions

In this work, we develop an MISOCP model using conic optimization that can allow data variation over the network by setting a single parameter. The proposed model reduces energy consumption in communication networks remarkably compared to the existing approach and the model shows it's outstanding performance in larger networks.

We utilize the idea of an ellipsoidal uncertainty set to allow data variation in the network and each pair able to fluctuate independently. The power is minimized by shutting down useless and underutilized links in the network. The proposed model can shut down more links compared to the existing works.

The MISOCP model is proposed as a robust counterpart of the Pipe model in association with the ellipsoidal uncertainty set. Though MISOCP is NP-hard, the proposed model can be run using modern software tools within a reasonable time. The proposed model also has achievements in terms of computing time correlated to the current approaches, which makes it a preferred choice in cases where computational time is a remarkable concern.

The rest of the sections of this work is designed as bellow: Section II starts with the development of the *green Pipe* model approach [6] following the presentation of the network and power models. In addition, the formulation of the green Hose approach is also presented in this section. The proposed green Ellipsoid model approach is developed in Section II-E after presenting the ellipsoidal uncertainty set. The numerical simulations with experiments are presented in Section III. The presentation of the achievements of the green Ellipsoid model can be found in this section. In Section IV, the article concludes with final remarks and outlines the directions for further work.

II. Model Formulation

A. Formulation of Green Pipe Model

To formulate the proposed MISOCP model, we use a model as a reference named the green Pipe model introduced by Bianzino et al. [6]. The authors formulated the green Pipe model as a mixed-integer optimization problem where the data demand for each pair is known in advance.

The detail of the green Pipe model approach is described as below:

$$\min \frac{1}{2} \sum_{(i,j)\in A} \left(\frac{u_{ij}+u_{ji}}{c_{ij}} P_{fij} + b_{ij} P_{0ij} \right) \tag{1a}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 1, \quad \forall (p,q)\in W, i=p, \tag{1b}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 0, \quad \forall (p,q)\in W, \forall i \in V\setminus\{p,q\}, \tag{1c}$$

$$\sum_{(p,q)\in W} d_{pq} x_{ij}^{pq} = u_{ij}, \qquad \forall (i,j)\in A, \tag{1d}$$

$$u_{ij} \le c_{ij}, \qquad \forall (i,j)\in A, \tag{1e}$$

$$N b_{ij} \ge u_{ij} + u_{ji}, \qquad \forall (i,j)\in A, \tag{1f}$$

$$x_{ij}^{pq} \ge 0, \qquad \forall (p,q)\in W, \forall (i,j)\in A, \tag{1g}$$

$$b_{ij} \in \{0,1\}, \qquad \forall (i,j)\in A. \tag{1h}$$

The variable, x_{ij}^{pq} is the portion of data passing from node p to q through the link (i,j). The constraint (1b) represents that the total portions of data broadcasting from node $i(=p)$ is equal to 1. The equations (1b) and (1c) indicate the network data stream management conditions. The constraint (1c) demonstrates that if the node i is neither a source nor a destination node, the total segment of data entering to node i must be the same as that of passing from node i. The equation (1d) represents the total data stream over link $(i,j)\in A$ and the constraint (1e) indicates the link load condition. If there is some data in one direction, the constraint (1f) is applied to keep the link on. The number, N is taken to be at least two times bigger than the maximal load capacity of the link and considered as a positive number. The statement, (1a) reduces the network energy utilization by shutting down a few useless links. We consider that the amount of c_{ij} and c_{ji} are equal in both directions, i.e. link capacity for both directions is the same. To avoid measuring the energy utilization twice, the objective function (1a) is divided by 2 for each link.

B. Power Model

The following power model is applied to formulate the proposed model:

$$PC = (P_M - P_0)x + P_0, \tag{2}$$

where, PC is the total power consumption when the link is kept on, otherwise 0, P_M is the maximal power when the link is utilized in its packed load, P_0 is the mandatory power to keep the link on, x represents the segment of data passing through the link, and definitely $0 \le x \le 1$. When the link is utilized to its packed load, the power consumption progressively increases from P_0 to P_M, which we call *energy aware* model in our case. If $P_M = P_0$ the approach is known as *energy agnostic* and if $P_0 = 0$ it is named as *fully proportional*. The considered power model is described in the following Figure 1 used by Bianzino et al. [6].

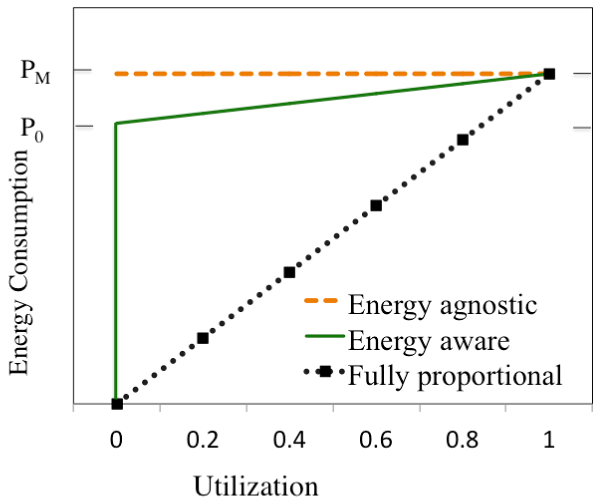


Fig. 1. Power model used in Bianzino et al. [Source: redesigned from [6]].

C. Network Model

A network represented by $G(V,A)$ is a directed graph, where A is the set of links and V is the set of nodes. A network with the considered components is described in Figure 2, where Q is the set of edge nodes and $Q \subseteq V$.

Network data is entering into and passing outside the network using the edge nodes. In this research, the set of edge-node pairs is expressed as W and $(p, q) \in W$, where $p \neq q$ and $p \in Q$ and $q \in Q$. The full duplex links are considered in this work. b_{ij} is a binary variable that indicates the on/off situation of the link. Here u_{ij} and c_{ij} represent the data stream and capacity on link $(i, j) \in A$, respectively. P_{fij} is the slope of the affine function of the power model and P_{0ij} is a constant. The power utilization of a link (i, j) is equal to P_{0ij} when it is kept on and does not flow any data.

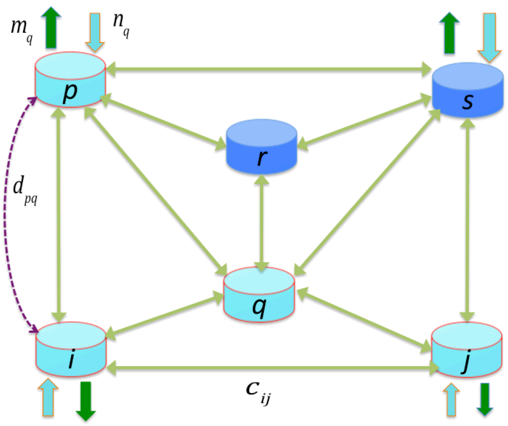


Fig. 2. Network model.

D. Development of Green Hose Approach

In the green Hose approach, the researchers think that the network operator can not examine the given data demand but can easily determine the total outgoing and incoming data. They described the total amount of outgoing data from node p as

$$\sum_{q} d_{pq} \leq m_p, \qquad \forall p \in Q, \tag{3}$$

here m_p is the maximal volume of data that node p can deliver to another node. The total volume of entering data to node q is represented by

$$\sum_{p} d_{pq} \leq n_q, \qquad \forall q \in Q, \tag{4}$$

here n_q is the maximal amount of data that node q can receive from another node.

Duffield et al. [23], Chu and Lea [24], and Liu and Guo [25] presented their data approaches using the total outgoing and incoming data bounds and named their work as the Hose model approach. In the Hose approach, there is an option for fluctuations of data and the uncertainty set for data is described by:

$$\mathcal{H} = \left\{ \mathbf{d} \in R^{|W|} : \begin{array}{ll} \sum_{q} d_{pq} \leq m_p, & \forall p \in Q, \\ \sum_{p} d_{pq} \leq n_q, & \forall q \in Q, \\ d_{pq} \geq 0, & \forall (p, q) \in W, \end{array} \right\}. \tag{5}$$

The final form of the green Hose model is represented as

$$\min \frac{1}{2} \sum_{(i,j) \in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} P_{fij} + b_{ij} P_{0ij} \right) \tag{6a}$$

$$\text{s.t.} \sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 1, \quad \forall (p, q) \in W, i = p, \tag{6b}$$

$$\sum_{j:(i,j) \in A} x_{ij}^{pq} - \sum_{j:(j,i) \in A} x_{ji}^{pq} = 0, \quad \forall (p, q) \in W, \forall i \in V \setminus \{p, q\}, \tag{6c}$$

$$\sum_{p \in Q} m_p \alpha_{ij}(p) + \sum_{p \in Q} n_p \beta_{ij}(p) = u_{ij}, \quad \forall (i, j) \in A, \tag{6d}$$

$$x_{ij}^{pq} \leq \alpha_{ij}(p) + \beta_{ij}(q), \quad \forall (i, j) \in A, \forall (p, q) \in W, \tag{6e}$$

$$\alpha_{ij}(p), \beta_{ij}(q) \geq 0, \quad \forall (i, j) \in A, \forall (p, q) \in W, \tag{6f}$$

$$u_{ij} \leq c_{ij}, \qquad \forall (i, j) \in A, \tag{6g}$$

$$N b_{ij} \geq u_{ij} + u_{ji}, \qquad \forall (i, j) \in A, \tag{6h}$$

$$x_{ij}^{pq} \geq 0, \quad \forall (p, q) \in W, \forall (i, j) \in A, \tag{6i}$$

$$b_{ij} \in \{0, 1\}, \qquad \forall (i, j) \in A. \tag{6j}$$

Here, $\alpha_{ij}(p)$ and $\beta_{ij}(p)$ are dual variables applied in the dual conversion. $\alpha_{ij}(p)$ denotes the ratio of data on link (i, j) outgoing from node p and $\beta_{ij}(p)$ represents the ratio of data on link (i, j) arriving at node p.

The green Hose model approach can fluctuate the data applying the hose uncertainty set, whereas the green Pipe model does not have any option for data fluctuations. But the achievement of the green Hose approach is much lesser than that of the green Pipe model.

In our current work, our aim is to articulate a MISOCP model applying the ellipsoidal uncertainty set, which can enhance the success of the green Hose model with an option for data fluctuations over the network.

E. Development of Proposed Green Ellipsoid Model

To develop the proposed MISOCP model, we usage the following ellipsoidal uncertainty set to permit data fluctuations over the network. The said uncertainty set is firstly applied by Das et al. [18] for minimizing congestion ratios in communication networks. In this work, we apply the ellipsoidal uncertainty set to green computing. In our case, we need the predicted data requirements $\bar{d}_{pq}$ for each $(p, q) \in W$ and the real data demands d_{pq} are contained in the ellipsoid. The total amount of squared data fluctuations is bound by a positive constant, η, which is the radius of the ellipsoid and indicates the total data variations in the network. The ellipsoidal uncertainty set is expressed by

$$\Omega_\eta = \left\{ \mathbf{d} : \sqrt{\sum_{(p,q) \in W} \delta_{pq} (d_{pq} - \bar{d}_{pq})^2} \leq \eta \right\}, \tag{7}$$

here the parameter $\delta_{pq} > 0$ for each $(p, q) \in W$ indicates the weight of pair (p, q). If there is no facts regarding data variation in advance, we can put $\delta_{pq} = 1$ for each $(p, q) \in W$.

To propose the MISOCP model, we use robust optimization approach to the green Pipe model. As the equation (1d) is valid for each $\mathbf{d}$, it should hold for max $\mathbf{d} \in \Omega_\eta$, i.e

$$\max_{\mathbf{d}\in\Omega_\eta} \left(\sum_{(p,q)\in W} d_{pq} x_{ij}^{pq} \right) = u_{ij}, \quad \forall (i,j) \in A. \tag{8}$$

We introduced a variable v_{pq} for every $(p,q) \in W$ such that

$$v_{pq} = \sqrt{\delta_{pq}}(d_{pq} - \bar{d}_{pq}), \tag{9}$$

where v_{pq} represents the data fluctuations between nodes p and q satisfying

$$\sqrt{\sum_{(p,q)\in W} v_{pq}^2} \leq \eta. \tag{10}$$

Therefore, the equation (1d) can be rewrite as

$$\sum_{(p,q)\in W} v_{pq} \frac{x_{ij}^{pq}}{\sqrt{\delta_{pq}}} \leq u_{ij} - \sum_{(p,q)\in W} \bar{d}_{pq} x_{ij}^{pq}, \quad \forall (i,j) \in A, \tag{11}$$

which is valid $\forall \mathbf{v} \in R^{|W|}$ such that $||\mathbf{v}|| \leq \eta$.

Now, let

$$\Theta_\eta = \left\{ \mathbf{v} \in R^{|W|} : ||\mathbf{v}|| \leq \eta \right\}, \tag{12}$$

which is a closed ball. For all $\mathbf{v} \in R^{|W|}$, we have

$$\mathbf{x}_{ij}^T \mathbf{v} \leq u_{ij} - \sum_{(p,q)\in W} \bar{d}_{pq} x_{ij}^{pq}, \quad \forall (i,j) \in A. \tag{13}$$

If the equation (13) holds $\forall \mathbf{v} \in R^{|W|}$, it should holds for max $\mathbf{v} \in \Theta_\eta$, i.e.

$$\max_{\mathbf{v}\in\Theta_\eta} \mathbf{x}_{ij}^T \mathbf{v} \leq u_{ij} - \sum_{(p,q)\in W} \bar{d}_{pq} x_{ij}^{pq}, \quad \forall (i,j) \in A. \tag{14}$$

Lemma 1 is used to measure the value of the left-hand side of the equation (14).

Lemma 1: If $\Gamma_\epsilon = \{\mathbf{x} \in R^n : ||\mathbf{x}|| \leq \epsilon\}$, where $||\cdot||$ is the Euclidean norm. then for any $\epsilon > 0$ and for given $\mathbf{a} \in R^n$, we have

$$\max_{\mathbf{x}\in\Gamma_\epsilon} \mathbf{a}^T \mathbf{x} = \epsilon ||\mathbf{a}||.$$

The learners are requested to consult with the work done by Das et al. [26] for proof of the Lemma 1. Using the Lemma 1, the equation (14) can be written as

$$\eta \sqrt{\sum_{(p,q)\in W} \frac{(x_{ij}^{pq})^2}{\delta_{pq}}} \leq u_{ij} - \sum_{(p,q)\in W} \bar{d}_{pq} x_{ij}^{pq}, \quad \forall (i,j) \in A, \tag{15}$$

which represents a second-order cone (SOC) constraint since left-hand side is a Euclidean norm and right-hand side is a constant term. Replacing the data flow constraint (1d) in the green Pipe model by the SOC constraint (15), we formulate proposed MISOCP model as follows:

$$\min \frac{1}{2} \sum_{(i,j)\in A} \left(\frac{u_{ij} + u_{ji}}{c_{ij}} P_{fij} + b_{ij} P_{0ij} \right) \tag{16a}$$

$$\text{s.t.} \sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 1, \quad \forall (p,q) \in W, i = p, \tag{16b}$$

$$\sum_{j:(i,j)\in A} x_{ij}^{pq} - \sum_{j:(j,i)\in A} x_{ji}^{pq} = 0, \quad \forall (p,q) \in W, \forall i \in V \setminus \{p,q\}, \tag{16c}$$

$$\sqrt{\sum_{(p,q)\in W} \frac{(x_{ij}^{pq})^2}{\delta_{pq}}} \leq \frac{1}{\eta} \left(u_{ij} - \sum_{(p,q)\in W} \bar{d}_{pq} x_{ij}^{pq} \right), \quad \forall (i,j) \in A, \tag{16d}$$

$$u_{ij} \leq c_{ij}, \quad \forall (i,j) \in A, \tag{16e}$$

$$N b_{ij} \geq u_{ij} + u_{ji}, \quad \forall (i,j) \in A, \tag{16f}$$

$$x_{ij}^{pq} \geq 0, \quad \forall (p,q) \in W, \forall (i,j) \in A, \tag{16g}$$

$$b_{ij} \in \{0,1\}, \quad \forall (i,j) \in A. \tag{16h}$$

The model expressed by (16) for green computing represents an MISOCP model since the constraint (16d) is a second-order cone constraint and it involves the binary variable b_{ij}, which indicates the on-off situation of links. We named this model as a green ellipsoid model since an ellipsoidal uncertainty set is used to develop this model. Though MISOCP is an NP-hard problem, the proposed model can be run using optimization tools within a reasonable computation time.

III. Numerical Experiment

A. Experiment Settings

To prove the achievement of the proposed model numerically, we conduct some experiments considering the backbone networks used by Ouédraogo and Oki [17], Oki and Iwaki [27], and Das et al. [18] (Network 1, Network 2, and Network 3). Figure 3 describes the details of the networks used in the experiments. The performances of the green Ellipsoid (G. Ellipsoid) approach are measured with the green Hose (G. Hose) and green Pipe (G. Pipe) approaches. The estimated data demands, $\bar{d}_{pq}$ that are needed for the G. Pipe model are arbitrarily taken with a homogeneous sharing in the length of (1,150). The capacities of the considered links, c_{ij} are chosen in the range of (800, 2800). The values of P_{fij} and P_{0ij} are determined as 0.3 and 1.7, respectively from Bianzino et al. [6] for 1 Gbps links.

The consumed energy savings are evaluated using the following equation for each model:

$$\text{power savings ratio} = \frac{E_{\text{T}} - E_{\text{R}}}{E_{\text{T}}}, \tag{17}$$

here E_{T} is the total consumed energy when all links keep on and E_{R} is the requisite power after the optimal result of the considered case. The values of m_p and n_q are calculated using: $m_p = \sum_{q\in Q} \bar{d}_{pq}$, $\forall p \in Q$ and $n_q = \sum_{p\in Q} \bar{d}_{pq}$, $\forall q \in Q$.

A Windows-based computer with an Intel(R) Core(TM) i7-4790 CPU @ 3.60 GHz and 16 GB of memory is used to conduct the numerical experiments. The Python programming language is used to code the programs. The considered problems are solved by using the professional optimization software, Gurobi (version 9.1.2) [28].

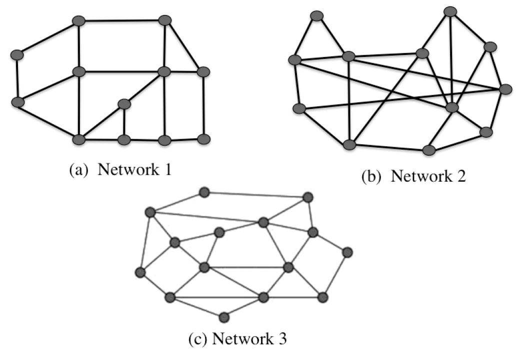


Fig. 3. Sampling networks.

TABLE I
COMPONENTS OF SAMPLING NETWORKS DESCRIBED IN FIGURE 3.

Network	No. of nodes considered	No. of links considered
Network 1	12	18
Network 2	12	22
Network 3	15	27

B. Energy Savings Achieved by G. Hose and G. Pipe Models

As we discussed earlier, the G. Pipe approach has no margin for data fluctuations due to the fixed amount of traffic. But the G. Hose model has the opportunity for fluctuations due to the wide range of hose uncertainty set. In this section, our goal is to show the comparative performances of the G. Pipe and G. Hose models numerically.

Figure 4 describes that the G. Pipe approach saves 5% more power than the G. Hose approach in Network 2, while in Network 1, the G. Pipe model saves 11% more energy than the G. Hose model, and this figure varies from more than 14% energy saving in Network 3. This experiment demonstrates that the G. Pipe approach works better in larger networks in terms of energy savings. The reason is that the G. Hose approach has a large dimension of data variation for fluctuations that require extra energy, which we addressed in the theory.

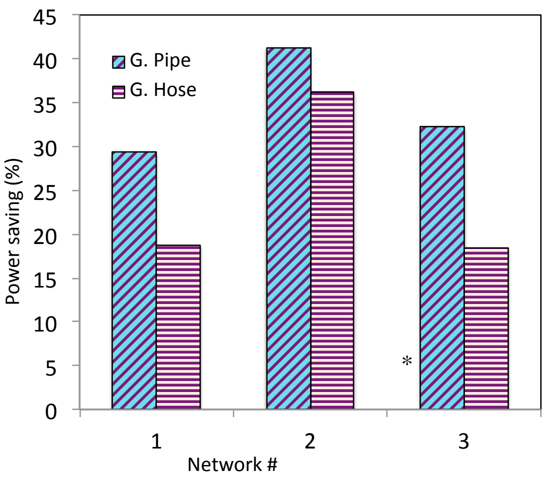


Fig. 4. Achievement of G. Hose and G. Pipe models in terms of power savings.

C. Optimal Value of Green Ellipsoid Model for Different Values of η

The optimal result of the green Ellipsoid (G. Ellipsoid) model is compared between the considered networks for different values of η, which are described in Figure 5. The value of η indicates the total volume of traffic variations in the backbone network.

Figure 5 indicates that when the values of η are comparatively larger, the G. Ellipsoid model demonstrates better performance in Networks 1, 2, and 3. Figure 5(a) shows that when the values of η vary from 1 to 10, the G. Ellipsoid model minimizes the power saving from 24.67% to 24.81% in Network 1, 24.23% to 25.42% in Network 2, and 24.02% to 26.45% in Network 3. But in Figure 5(b), when the values of η vary from 20 to 100 the G. Ellipsoid model minimizes the power saving from 24.97% to 29.41% in Network 1, 24.45% to 25.82% in Network 2, and 26.45% to 28.23% in Network 3.

Figure 5(c) shows that the G. Ellipsoid model gives the infeasible solution for $\eta = 400$ and above in Network 1. The proposed model also provides an infeasible solution for $\eta = 750$ and above in Network 2 and $\eta = 850$ and above in Network 3, respectively. This is because there are some constraints or limitations in the model's achievement due to setting the values of the parameters in the considered network structures.

D. Comparisons of Achievements Between the Models

In this section, the achievements of the G. Ellipsoid model are analyzed with the G. Hose and G. Pipe models for different values of η, which are demonstrated in Figures 6, 7, and 8 for Networks 1, 2, and 3, respectively.

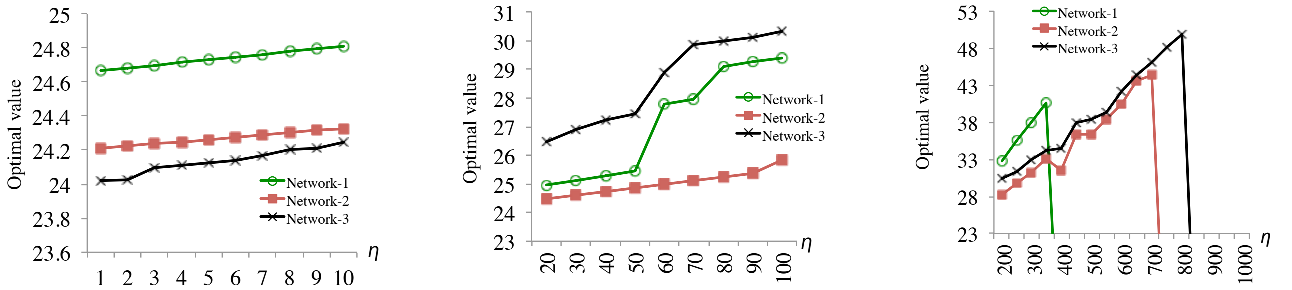
Figure 6 indicates that the G. Ellipsoid model minimizes the power consumption by 24.61%, 24.67%, and 29.41% for $\eta = 1$, $\eta = 10$, and $\eta = 100$, respectively, in Network 1, whereas this value is 24.65% for the G. Pipe model and 24.97% for the G. Hose model.

In Network 2, the G. Ellipsoid model can save 24.21%, 25.42%, and 26.82% of power consumption for $\eta = 1$, $\eta = 10$, and $\eta = 100$, respectively, whereas the G. Pipe model can save 24.20% and the G. Hose model can save only 24.54% power utilization, which is described in Figure 7.

The interesting thing that happened in Network 3 is illustrated in Figure 8. The figure demonstrates that the G. Ellipsoid model performs lower than that of the G. Hose model for $\eta = 1$, but for $\eta = 10$ and $\eta = 100$, the proposed model shows outstanding performance in this larger network.

It is obvious that the achievement of the presented G. Ellipsoid model is almost close to the G. Pipe and G. Hose models for a smaller value of η, but for a higher value of η, the G. Ellipsoid model can save relatively more power consumption compared to the G. Pipe and G. Hose models, which is the advantage of the G. Ellipsoid model.

Figure 8 depicts that the G. Ellipsoid model can save 26.45% and 30.34% power consumption for $\eta = 10$ and $\eta = 100$, respectively, in Network 3, whereas the G. pipe and G. Hose models can save only 24.56% and 24.54%, respectively, in the same network. This experiment demonstrates that the proposed model performs remarkably better in larger networks and also for a large amount of data fluctuation over



(a) Optimal value for η = 1~10 (b) Optimal value for η = 20~100 (c) Optimal value for η = 200~1000

Fig. 5. Comparisons of optimal values for different values of η for G. Ellipsoid model.

the backbone network, which is a significant achievement of the proposed model.

Another achievement of the G. Ellipsoid model is that it has no boundary for fluctuations for each link, like the G. Hose model. This special shows that the G. Ellipsoid model attempts a more flexible and adaptable approach, allowing for smoother transitions and interactions within the network.

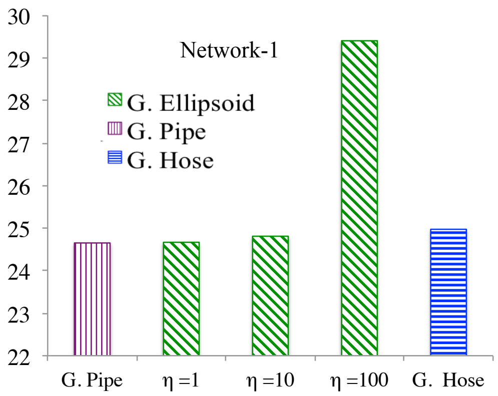


Fig. 6. Achievement of G. Ellipsoid model for Network 1.

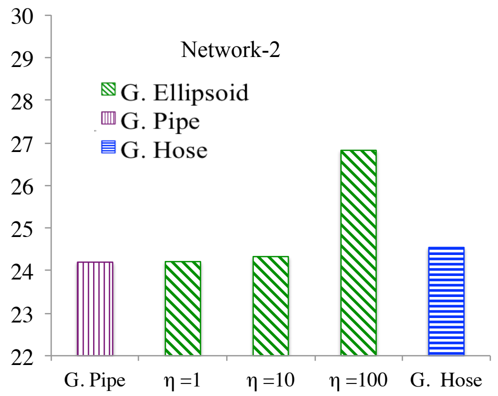


Fig. 7. Achievement of G. Ellipsoid model for Network 2.

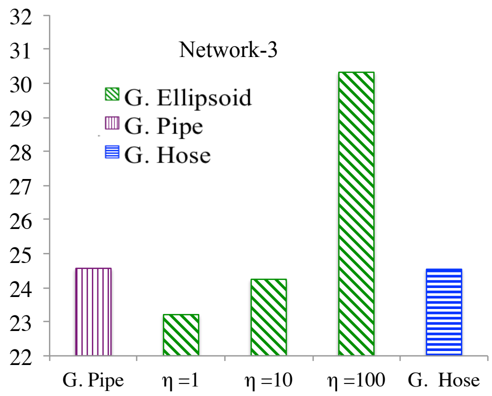


Fig. 8. Achievement of G. Ellipsoid model for Network 3.

E. Number of Deactivated Links

As we demonstrated earlier, the proposed G. Ellipsoid model minimizes the energy consumption by deactivating few unnecessary links in the networks. In this way, it can lead to significant power savings and improved overall efficiency by optimizing the network structure.

The comparison in terms of number of deactivated links is addressed in Table II for the considered Networks 1, 2, and 3. In the experiments, we consider $\eta = 10$ in the G. Ellipsoid model. This optical representation offers a concise and clear overview of how the different models perform in terms of deactivating unnecessary links in the networks.

The outcomes from the experiments depict that both the G. Pipe and G. Ellipsoid models can deactivate 6 links in Network 1, while the G. Hose model can deactivate only 4 links in the same network. In Network 2, the G. Hose model is able to save 9 links, whereas the G. Ellipsoid and G. Pipe models can save 10 links each in the same network. In Network 3, both the G. Ellipsoid and G. Pipe models are able to deactivate 11 links each, while the G. Hose model can deactivates only 7 links in the same network. This comparison again demonstrates that the G. Ellipsoid model outperforms the G. Hose model, as we described previously.

Figures 9, 10, and 11 also illustrate the success of the G. Ellipsoid model in terms of deactivating the links for higher values of η in the considered networks. The results indicate that the G. Ellipsoid model excels when higher values of η

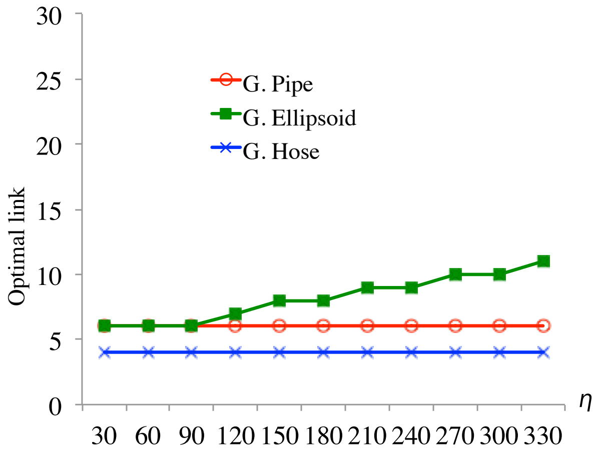


Fig. 9. Comparison of optimal link [Network 1].

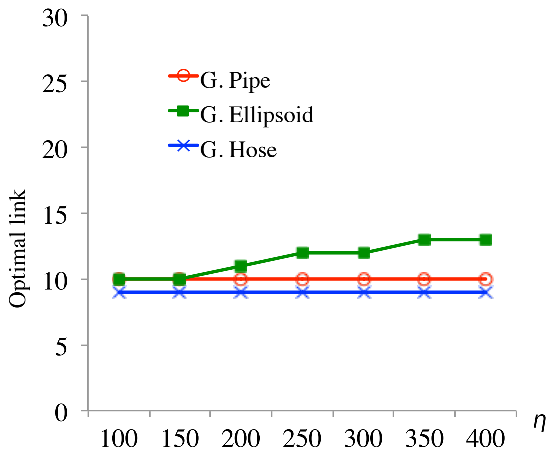


Fig. 10. Comparison of optimal link [Network 2].

are considered and the model exhibits superior performance in handling large network structures which is an additional achievement of the proposed model.

TABLE II
NUMBER OF DEACTIVATED LINKS FOR $\eta = 10$

Network name	Number of deactivated links		
	G. Pipe	G. Ellipsoid ($\eta = 10$)	G. Hose
Network 1	6	6	4
Network 2	10	10	9
Network 3	11	11	7

F. Comparisons of Computing Time Between the Models

The computing times for the examined models are addressed in Table III. The table depicts that the G. Ellipsoid model also has achievements in computing time in comparison to the G. Hose model for each of Networks 1, 2, and 3.

The G. Pipe model is designed to be simpler and faster compared to other models. It has the lowest number of constraints and variables, which means it is less complex to

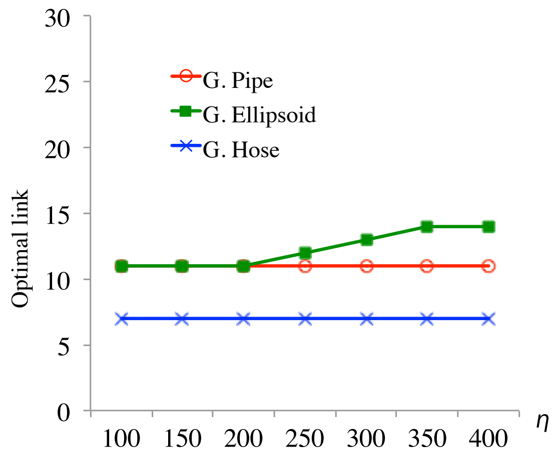


Fig. 11. Comparison of optimal link [Network 3].

TABLE III
COMPUTATION TIME FOR $\eta = 10$

Network name	Computing time [sec.]		
	G. Pipe	G. Ellipsoid	G. Hose
Network 1	61	531	1735
Network 2	80	750	2437
Network 3	153	2567	5797

solve computationally. Additionally, it does not include an uncertainty set, which further reduces the time required for solving the model. The G. Ellipsoid approach is faster than the G. Hose approach but slower than the G. Pipe approach in terms of computing time. Both the G. Ellipsoid model and the G. Hose model include uncertainty sets, but the G. Ellipsoid approach is rapid than the G. Hose model due to the difference in the sizes of their feasible regions.

The reasonable area of the G. Hose approach is indeed larger than that of the G. Ellipsoid model, meaning it includes more possible solutions. This increased complexity in the feasible region of the G. Hose model leads to a longer computation time compared to the G. Ellipsoid model. As a result, the G. Ellipsoid model is able to solve optimization problems more quickly and efficiently, which makes the G. Ellipsoid model a preferred choice in cases where computational time is a remarkable concern. The efficiency of the proposed G. Ellipsoid approach in terms of computing time can advance rapid decision-making and improve overall achievement in several applications.

IV. CONCLUSION

In this work, a mixed integer second-order cone programming (MISOCP) approach is proposed to minimize power savings in communication networks. The conic optimization technique is utilized to work out the proposed green Ellipsoid (G. Ellipsoid) model. The intention is to reduce network power while considering variations in network data using robust optimization. An ellipsoidal uncertainty set is employed to grant variations in network data.

The G. Ellipsoid model reduces total power consumption by shutting down unnecessary links across the network.

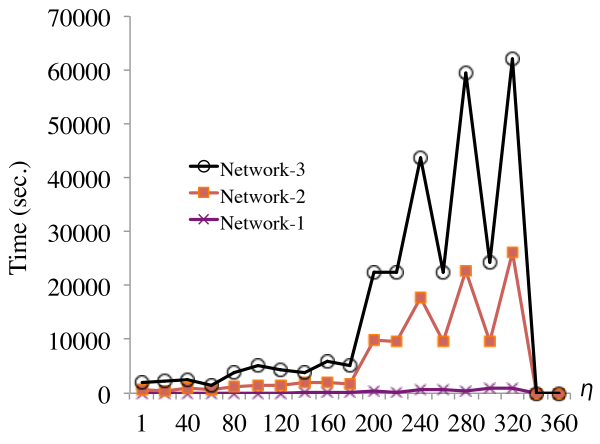


Fig. 12. Computation time of G.Ellipsoid model for different η.

Numerical results demonstrate that the introduced G. Ellipsoid model achieves greater power savings than the existing G. Hose model for different values of the parameter, η. The results also indicate that the presented model performs appreciably better in the larger networks and also for a big amount of data fluctuation over the backbone network, which is the significant achievement of the proposed model.

Since MISOCP is classified as an NP-hard problem, this study also proposes an improved method to minimize computation time, thus rendering the model tractable using optimization software tools. By addressing the challenge of computational complexity associated with MISOCP, this work enhances the practicality and contribution of power saving in communication networks compared to the G. Hose model.

To advance the achievement of the proposed model, further study can be conducted to introduce a heuristic for the MISOCP problem and more complex network topologies can be used to conduct numerical experiments.

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