The Application of Fuzzy Comprehensive Evaluation Method in the Evaluation of Teaching Quality of Secondary School Mathematics

Lili Wang, Linlin Zhang, Meng Hu

Abstract—**From the perspective of teachers' teaching, this paper proposes and establishes a two-level fuzzy comprehensive evaluation model for the first time on the basis of constructing a scientific evaluation index system for secondary school mathematics classroom teaching quality. The feasibility and effectiveness of the model are illustrated by an example, and the MATLAB programs for solving the model are also given. The interest of this paper is that the evaluation model can evaluate the effect of secondary school mathematics classroom teaching from multiple dimensions, and give a fast algorithm. The evaluation indexes given in this paper are more comprehensive, and the evaluation results are more valuable.**

Index Term--**fuzzy comprehensive evaluation; entropy method; membership degree; teaching quality evaluation**

I. Introduction

EDUCATION is the foundation of a hundred-year plan, and education is the foundation of a country, which directly contributes to the quantity and quality of a future generation of talents for a country. Meanwhile, it is the basis of a national development of innovative ability and scientific and technological level. The quality of education not only exerts an influence on the growth of children, but also affects the future development of a country. The objective of teaching evaluation is to motivate teachers, improve teaching methods, keep pace with the times, adapt to social progress. In the meantime, it aims to provide better assistance for students in their learning progress. The feedback obtained from teaching evaluation also enables parents to keep abreast of the learning situation of their children, and to help their children to make progress through the cooperation between the home and the school. Besides, the results of teaching evaluation can also be utilized as an indicator of the teaching level of a school [1]. For this reason, it is imperative to improve the evaluation system of mathematics classroom teaching, which is of great benefit to the development of society and the progress of human spiritual civilization.

1.1 On classroom evaluation in secondary schools

Classroom teaching evaluation is an essential method to improve the teaching quality, which is also the core content of the whole teaching evaluation system, with the role of motivational, diagnostic, orienting and supervisory. It represents a significant part of classroom teaching, which runs through each aspect of teaching activities, with scientific evaluation of classroom teaching making the classroom more vitalized [2].

At the present time, there are still some issues in the mathematics classroom teaching in junior and senior high schools. For instance, the attention of students in class is not concentrated enough, the mathematics classroom fails to achieve the expected results, and it is difficult to realize the teaching objectives. As time passes, the students become reluctant to mathematics, and even begin to be anorexic, which has a severe impact on the cultivation of talents in our country. Consequently, it is imperative to improve the teaching evaluation system, which not only impacts children's education and growth, but also concerns the future of a country's talent cultivation and construction.

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II. Theory of Fuzzy Comprehensive Evaluation Method

Fuzzy comprehensive evaluation is a comprehensive evaluation method based on fuzzy mathematics, which combines the membership degree theory with the traditional comprehensive evaluation theory to comprehensively evaluate the objects or systems affected by multiple factors [3]. According to the different evaluation factors, fuzzy comprehensive evaluation is divided into single-level fuzzy comprehensive evaluation and multi-level fuzzy comprehensive evaluation.

2.1 Steps of single-level fuzzy comprehensive evaluation method

(1) Determining the factor set U to be evaluated

$$U = \{u_1, u_2, ..., u_n\} .$$

The factors reflect the performance indicators and attributes of the evaluation objects.

(2) Determining the evaluation level set V

$$V = \{v_1, v_2, ..., v_m\} .$$

In essence, the evaluation level is a division of the change interval of the evaluation object. The number of evaluation levels should not be too small or too much, generally taken as five levels.

(3) Determining the weight vector of evaluation factors

Assigning a fuzzy vector w to the weight vector,

$$w = (w_1, w_2, \cdots, w_m),$$

where $w_i > 0$ denotes the weight of the i th factor, while w reflects the importance of each factor.

(4) Establishment of a fuzzy comprehensive evaluation matrix

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \cdots & r_{nm} \end{bmatrix},$$

where r_{ij} denotes the membership degree of an evaluated object belonging to the evaluation level v_j from the evaluation factor u_i, see [5].

(5) Selecting a fuzzy comprehensive evaluation model

There are five commonly used comprehensive evaluation models, which are as follows [6]:

① Model $M(\bullet,+)$

The component of the fuzzy comprehensive evaluation vector is $b_j=\sum_{i=1}^{n} w_i \bullet r_{ij} \, (j=1,2,\cdots,m)$.

Using MATLAB R2020b software to calculate, define the following function file prod_sum.m:

```
% Model M(•,+);
%w is the weight vector;
%R is the fuzzy comprehensive evaluation matrix;
function b=prod_sum(w,R)
b=w*R;
end
```

② Model $M(\wedge,\vee)$

The component of the fuzzy comprehensive evaluation vector is $b_j=\max_{1\le i\le n}\left\{\min(w_i,r_{ij})\right\}(j=1,2,\cdots,m)$.

Using MATLAB R2020b software to calculate, define the following function file min_max.m:

```
% Model  M(∧,∨);
%w is the weight vector;
%R is the fuzzy comprehensive evaluation matrix;
function b=min_max(w,R)
[m,s]=size(w);[s1,n]=size(R);
if s1~=s
    return;
end
for i=1:m
    for j=1:n
        b(i,j)=0;
        for k=1:s
            x=0;
            if w(i,k)<R(k,j)
                x=w(i,k);
            else x=R(k,j);
            end
            if b(i,j)<x
                b(i,j)=x;
            end
        end
    end
end
```

③ Model $M(\bullet,\vee)$

The component of the fuzzy comprehensive evaluation vector is

$$b_j=\max_{1\le i\le n}\left\{w_i \bullet r_{ij}\right\}(j=1,2,\cdots,m).$$

Using MATLAB R2020b software to calculate, define the following function file prod_max.m:

```
% Model  M(•,∨);
%w is the weight vector;
%R is the fuzzy comprehensive evaluation matrix;
function b=prod_max (w,R)
[m,s]=size(w);[s1,n]=size(R);
if s1~=s
    return;
end
for i=1:m
    for j=1:n
        b(i,j)=0;
        for k=1:s
            if b(i,j)<w(i,k)*R(k,j)
                b(i,j)=w(i,k)*R(k,j);
            end
        end
    end
end
```

④ Model $M(\wedge,\oplus)$

The component of the fuzzy comprehensive evaluation vector is

$$b_i=\min\left\{1,\sum_{i=1}^{n}\min(w_i,r_{ij})\right\}(j=1,2,\cdots,m).$$

Using MATLAB R2020b software to calculate, define the following function file min_bsum.m:

```
% Model  M(∧,⊕);
%w is the weight vector;
%R is the fuzzy comprehensive evaluation matrix;
function b= min_bsum (w,R)
[m,s]=size(w);[s1,n]=size(R);
if s1~=s
    return;
end
for i=1:m
    for j=1:n
        for k=1:s
            if w(i,k)< R(k,j);
                x(k,j)=w(i,k);
            else x(k,j)= R(k,j);
            end
            sum_x=sum(x);
            b(i,j)=min(1,sum_x(i,j));
            end
        end
    end
end
```

⑤ Model $M(\bullet,\oplus)$

The component of the fuzzy comprehensive evaluation vector is $b_i = \min\left\{1, \sum_{i=1}^{n}(w_i, r_{ij})\right\}(j = 1, 2, \cdots, m)$.

Using MATLAB R2020b software to calculate, define the following function file prod_bsum.m:

```
% Model M(•,⊕);
%w is the weight vector;
%R is the fuzzy comprehensive evaluation matrix;
function b=prod_bsum(w,R)
[m,s]=size(w);[s1,n]=size(R);
if s1~=s
    return;
end
for i=1:m
    for j=1:n
        for k=1:s
            x(k,j)=w(i,k)*R(k,j);
        end
        sum_x=sum(x);
        b(i,j)=min(1,sum_x(i,j));
    end
end
```

The above models are selected for the composite operation to derive the fuzzy comprehensive evaluation vector:

$$B = w \circ R = (b_1, b_2, \cdots, b_m).$$

(6) Determining the evaluation results in accordance with the fuzzy comprehensive evaluation vector

$$B = (b_1, b_2, \cdots, b_m).$$

2.2 Steps of multi-level fuzzy comprehensive evaluation method (taking two-level case as an example)

(1) Determining the first-level factor set

The factor set $U = \{u_1, u_2, \cdots, u_n\}$ is divided into groups to obtain $U = \{U_1, U_2, \cdots, U_k\}$,

$$U = \bigcup_{i=1}^{k} U_i,\ U_i \cap U_j = \varnothing\ (i \neq j),$$

and $U = \{U_1, U_2, \cdots, U_k\}$ is the first-level factor set.

(2) Determining the evaluation level set

The evaluation level set is assumed to be

$$V = \{v_1, v_2, \ldots\ldots, v_m\}.$$

(3) Determining the second-level fuzzy comprehensive evaluation matrices

Based on the second-level factor set,

$$U_i = \left\{u_1^{(i)}, u_2^{(i)}, \cdots, u_{n_i}^{(i)}\right\},$$

the single-level fuzzy comprehensive evaluation method is used for evaluation, and the fuzzy comprehensive evaluation matrix is obtained:

$$R_i = \begin{bmatrix} r_{11}^{(i)} & r_{12}^{(i)} & \cdots & r_{1m}^{(i)} \\ r_{21}^{(i)} & r_{22}^{(i)} & \cdots & r_{2m}^{(i)} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n_i 1}^{(i)} & r_{n_i 2}^{(i)} & \cdots & r_{n_i m}^{(i)} \end{bmatrix}, i = 1, 2, \cdots, k.$$

(4) Determining the weight vector of the second-level evaluation factors

The weight of

$$U_i = \left\{u_1^{(i)}, u_2^{(i)}, \cdots, u_{n_i}^{(i)}\right\}$$

is determined to be

$$w_i = \left\{w_1^{(i)}, w_2^{(i)}, \cdots, w_{n_i}^{(i)}\right\}, i = 1, 2, \cdots, k.$$

(5) Determining the second-level fuzzy comprehensive evaluation vector

The second-level fuzzy comprehensive evaluation vector is obtained by a composite operation between w_i and R_i, and

$$B_i = w_i \circ R_i, i = 1, 2, \cdots, k.$$

(6) Determining the fuzzy comprehensive evaluation matrix of the first-level factors

The fuzzy comprehensive evaluation matrix for the first-level factor set

$$U = \{U_1, U_2, \cdots, U_k\}$$

is constructed by the second-level fuzzy comprehensive evaluation vector, that is,

$$R = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_k \end{pmatrix}.$$

(7) Fuzzy comprehensive evaluation of the first-level factors

The weight of the first-level factor set

$$U = \{U_1, U_2, \cdots, U_k\}$$

is determined to be $w = \{w_1, w_2, \cdots, w_k\}$. The first-level fuzzy comprehensive evaluation vector is obtained by a composite operation of w and R, and

$$B = w \circ R.$$

(8) Determining the evaluation results based on the fuzzy comprehensive evaluation vector B.

If each sub-factor set $U_i\ (i = 1, 2, \cdots, k)$ contains more factors, it is possible to subdivide U_i and more levels of fuzzy comprehensive evaluation can be performed [5].

III. APPLICATION OF FUZZY COMPREHENSIVE EVALUATION METHOD IN CLASSROOM TEACHING EVALUATION

3.1 Determining the factor set of evaluation objects

In the teaching evaluation of teachers, this paper selects five factors of teaching literacy, teaching contents, teaching methods, teaching attitude and teaching effects as the first level factors, and set another 20 factors as the second-level factors, specifically shown in Table 1.

Evaluation factor sets are established:

First-level factor set:

$$U = \{A, B, C, D, E\}.$$

Second-level factor sets:

A={a1,a2,a3,a4 }; B={b1,b2,b3,b4};
C={c1,c2,c3,c4 }; D={d1,d2,d3,d4};
E={e1,e2,e3,e4 }.

In this paper, a total of 10 mathematics teachers in the school are selected as the evaluation objects. After listening to the open class of each teacher, a questionnaire is adopted to rate each factor in the second-level factors, respectively. The full score of each factor is 10 points, and statistics are conducted, with the score taken as the average and rounded, and the specific data are shown in Table 2.

3.2 Determining the evaluation set of evaluated objects

TABLE 3 RATING CRITERIA FOR QUANTITATIVE EVALUATION

Hierarchy	Evaluation value
Excellent	9~10
Good	8~9
Fair	7~8
Pass	6~7
Fail	0~6

3.3 Determining the weighting coefficients via entropy method

Information entropy is a measure of information uncertainty, the greater the amount of information the smaller the entropy value, the greater the information utility value. The entropy method determines the weights by calculating the information entropy of each indicator observation [4].

3.3.1 Determination of the weights of the first-level factors

According to the influence of the relative change degree of the five first-level evaluation factors of teaching literacy, teaching contents, teaching methods, teaching attitude and teaching effects on the whole system, the weight coefficient is determined. The first-level factor evaluation score is obtained by summing the second-level factor evaluation scores. The specific data are shown in Table 4.

The observed values of the five first-level factors for the 10 teachers are $x_{ij} \geq 0$ $(i=1,2,\cdots,10; j=1,2,\cdots,5)$. The entropy method is applied below to determine the weights of the factors.

(1) Calculating the characteristic weight of each factor

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{10} x_{ij}}, (i=1,2,\cdots,10; j=1,2,\cdots,5).$$

(2) Calculating the entropy of each factor

$$e_j = -k\sum_{i=1}^{10} p_{ij} \ln(p_{ij}), (j=1,2,\cdots,5),$$

where $k = \frac{1}{\ln 10}$.

(3) Calculating the coefficient of variation for each factor

$$g_j = 1 - e_j, (j=1,2,\cdots,5).$$

(4) Determining the weighting coefficients of the factors

$$w_j = \frac{g_j}{\sum_{j=1}^{5} g_j}, (j=1,2,\cdots,5).$$

According to the entropy method [7], the weight of the first-level factors is calculated to be

$$w = (0.2, \ \ 0.1, \ \ 0.2, \ \ 0.2, \ \ 0.3).$$

3.3.2 Determination of the weights of the second-level factors

With reference to the calculation process of the weights of the first-level factors, the entropy value method is applied, and the weights of the second-level factors are obtained in the same way on the basis of the data in Table 2:

$$w_A = (0.2, \ \ 0.3, \ \ 0.3, \ \ 0.2),$$
$$w_B = (0.3, \ \ 0.2, \ \ 0.2, \ \ 0.3),$$
$$w_C = (0.1, \ \ 0.2, \ \ 0.3, \ \ 0.4),$$
$$w_D = (0.3, \ \ 0.2, \ \ 0.2, \ \ 0.3),$$
$$w_E = (0.2, \ \ 0.4, \ \ 0.,1 \ \ 0.3).$$

3.4 Determining the fuzzy comprehensive evaluation matrix

According to the influence of each evaluation factor on the teaching quality, the corresponding membership function is constructed to determine the fuzzy comprehensive evaluation matrix. Based on the data in Table 3, the membership functions are constructed by trapezoidal distribution method:

(1) Membership function for "Excellent" level

$$r_1 = \begin{cases} 0, & x \leq 8; \\ \frac{x-8}{9-8}, & 8 < x < 9; \\ 1, & x \geq 9. \end{cases}$$

(2) Membership function for "Good" level

$$r_2 = \begin{cases} 0, & x \leq 7; \\ \frac{x-7}{8-7}, & 7 < x \leq 8; \\ \frac{9-x}{9-8}, & 8 < x < 9; \\ 0, & x \geq 9. \end{cases}$$

(3) Membership function for "Fair" level

$$r_3 = \begin{cases} 0, & x \leq 6; \\ \frac{x-6}{7-6}, & 6 < x \leq 7; \\ \frac{8-x}{8-7}, & 7 < x < 8; \\ 0, & x \geq 8. \end{cases}$$

(4) Membership function for "Pass" level

$$r_4 = \begin{cases} 0, & x < 6; \\ \dfrac{7-x}{7-6}, & 6 \le x < 7; \\ 0, & x \ge 7. \end{cases}$$

(5) Membership function for “Fail” level

$$r_5 = \begin{cases} 1, & x < 6; \\ 0, & x \ge 6. \end{cases}$$

On the basis of the observed values of each factor in Table 2 and the corresponding membership functions, the membership degree is calculated, so as to determine the fuzzy comprehensive evaluation matrix.

Using MATLAB R2020b software to calculate, we can get the fuzzy comprehensive evaluation matrices for 10 teachers under the second-level factors.

Now, we calculate the fuzzy comprehensive evaluation matrices for 10 teachers under the second-level factors *a1,a2,a3,a4* as an example, the calculation program is as follows:

```
% calculate R_A (Ra)
clc;clear;
A=[8.5 9.0 9.4 9.7 8.9 8.6 7.8 9.1 8.5 9.0
   8.7 8.6 8.5 8.6 8.6 8.9 9.0 9.0 9.0 9.0
   8.9 8.9 8.6 8.7 7.9 9.0 9.7 9.5 9.6 9.5
   9.8 7.8 8.8 8.9 8.9 8.5 9.6 9.5 9.4 9.1];
[m,n]=size(A);
for j=1:n
    for i=1:m
        if A(i,j)<=8
            Ra(i,1,j)=0;
        elseif A(i,j)>8&A(i,j)<9
            Ra(i,1,j)=(A(i,j)-8)/(9-8);
        else Ra(i,1,j)=1;
        end
    end
    for i=1:m
        if A(i,j)<=7
            Ra(i,2,j)=0;
        elseif A(i,j)>7&A(i,j)<=8
            Ra(i,2,j)=(A(i,j)-7)/(8-7);
        elseif A(i,j)>8&A(i,j)<9
            Ra(i,2,j)=(9-A(i,j))/(9-8);
        else Ra(i,2,j)=0;
        end
    end
    for i=1:m
        if A(i,j)<=6
            Ra(i,3,j)=0;
        elseif A(i,j)>6&A(i,j)<=7
            Ra(i,3,j)=(A(i,j)-6)/(7-6);
        elseif A(i,j)>7&A(i,j)<8
            Ra(i,3,j)=(8-A(i,j))/(8-7);
        else Ra(i,3,j)=0;
        end
    end
    for i=1:m
        if A(i,j)<=6
            Ra(i,4,j)=0;
        elseif A(i,j)>6&A(i,j)<=7
            Ra(i,4,j)=(7-A(i,j))/(7-6);
        else Ra(i,4,j)=0;
        end
    end
    for i=1:m
        if A(i,j)<=6
            Ra(i,5,j)=1;
        else Ra(i,5,j)=0;
        end
    end
end
Ra
```

and

$$R_A(1) = \begin{bmatrix} 0.5000 & 0.5000 & 0 & 0 & 0 \\ 0.7000 & 0.3000 & 0 & 0 & 0 \\ 0.9000 & 0.1000 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(2) = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0.6000 & 0.4000 & 0 & 0 & 0 \\ 0.9000 & 0.1000 & 0 & 0 & 0 \\ 0 & 0.8000 & 0.2000 & 0 & 0 \end{bmatrix},$$

$$R_A(3) = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0.5000 & 0.5000 & 0 & 0 & 0 \\ 0.6000 & 0.4000 & 0 & 0 & 0 \\ 0.8000 & 0.2000 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(4) = \begin{bmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0.6000 & 0.4000 & 0 & 0 & 0 \\ 0.7000 & 0.3000 & 0 & 0 & 0 \\ 0.9000 & 0.1000 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(5) = \begin{bmatrix} 0.9000 & 0.1000 & 0 & 0 & 0 \\ 0.6000 & 0.4000 & 0 & 0 & 0 \\ 0 & 0.9000 & 0.1000 & 0 & 0 \\ 0.9000 & 0.1000 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(6) = \begin{bmatrix} 0.6000 & 0.4000 & 0 & 0 & 0 \\ 0.9000 & 0.1000 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 0.5000 & 0.5000 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(7) = \begin{bmatrix} 0 & 0.8000 & 0.2000 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R_A(8)=\begin{bmatrix}1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$R_A(9)=\begin{bmatrix}0.5000 & 0.5000 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$R_A(10)=\begin{bmatrix}1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0\end{bmatrix}.$$

3.5 Determining the fuzzy comprehensive evaluation model and calculating the fuzzy comprehensive evaluation vector

The five models which have been defined in section II can be used to calculate the fuzzy comprehensive evaluation vector, but one should select an effective model according to the calculation results, that is, the discrimination of each component of the fuzzy comprehensive evaluation vector is as large as possible. In this paper, the model $M(\bullet,+)$ is selected as an example, and the fuzzy comprehensive evaluation vectors of 10 teachers under the second-level factors *a1,a2,a3,a4 are*:

$$Z_A(1)=w_A \bullet R_A(1);$$
$$Z_A(2)=w_A \bullet R_A(2);$$
$$\cdots\cdots$$
$$Z_A(10)=w_A \bullet R_A(10).$$

Using MATLAB R2020b software to calculate, we only calculate Z_A as an example, the calculation program is as follows:

```
% using the Model M(•,+) to calculate Z_A (Za);
% calling the function file prod_sum.m;
clc;clear;
w=[0.2 0.3 0.3 0.2];
for j=1:10
        Za(:,:,j)=prod_sum(w,Ra(:,:,j));
end
Za
```

and

$Z_A(1)=$
$[0.7800 \quad 0.2200 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(2)=$
$[0.6500 \quad 0.3100 \quad 0.0400 \quad 0.0000 \quad 0.0000]$,
$Z_A(3)=$
$[0.6900 \quad 0.3100 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(4)=$
$[0.7700 \quad 0.2300 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(5)=$
$[0.5400 \quad 0.4300 \quad 0.0300 \quad 0.0000 \quad 0.0000]$,
$Z_A(6)=$
$[0.7900 \quad 0.2100 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(7)=$
$[0.8000 \quad 0.1600 \quad 0.0400 \quad 0.0000 \quad 0.0000]$,
$Z_A(8)=$
$[1.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(9)=$
$[0.9000 \quad 0.1000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_A(10)=$
$[1.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$.

In the same way, the fuzzy comprehensive evaluation vectors of 10 teachers under the second-level factors *b1,b2,b3,b4 are*:

$Z_B(1)=$
$[0.5500 \quad 0.4200 \quad 0.0300 \quad 0.0000 \quad 0.0000]$,
$Z_B(2)=$
$[0.4000 \quad 0.5700 \quad 0.0300 \quad 0.0000 \quad 0.0000]$,
$Z_B(3)=$
$[0.1200 \quad 0.8800 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_B(4)=$
$[0.7000 \quad 0.3000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_B(5)=$
$[0.6500 \quad 0.3300 \quad 0.3000 \quad 0.0000 \quad 0.0000]$,
$Z_B(6)=$
$[0.4800 \quad 0.4900 \quad 0.3000 \quad 0.0000 \quad 0.0000]$,
$Z_B(7)=$
$[0.3800 \quad 0.6000 \quad 0.2000 \quad 0.0000 \quad 0.0000]$,
$Z_B(8)=$
$[0.5400 \quad 0.4400 \quad 0.2000 \quad 0.0000 \quad 0.0000]$,
$Z_B(9)=$

$[0.2000 \quad 0.9500 \quad 0.3000 \quad 0.0000 \quad 0.0000]$,
$Z_B(10)=$
$[1.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$.

Fuzzy comprehensive evaluation vectors for 10 teachers under the second-level factors *c1,c2,c3,c4 are*:

$Z_C(1)=$
$[0.3000 \quad 0.6300 \quad 0.0700 \quad 0.0000 \quad 0.0000]$,
$Z_C(2)=$
$[0.7600 \quad 0.2400 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_C(3)=$
$[0.7600 \quad 0.2400 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_C(4)=$
$[0.5000 \quad 0.4900 \quad 0.0100 \quad 0.0000 \quad 0.0000]$,
$Z_C(5)=$
$[0.6200 \quad 0.3600 \quad 0.0200 \quad 0.0000 \quad 0.0000]$,
$Z_C(6)=$
$[0.6100 \quad 0.3300 \quad 0.0600 \quad 0.0000 \quad 0.0000]$,
$Z_C(7)=$
$[0.7300 \quad 0.2700 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_C(8)=$
$[0.5700 \quad 0.4300 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_C(9)=$
$[0.4300 \quad 0.5700 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_C(10)=$
$[1.7700 \quad 0.2300 \quad 0.0000 \quad 0.0000 \quad 0.0000]$.

Fuzzy comprehensive evaluation vectors for 10 teachers under the second-level factors *d1,d2,d3,d4 are*:

$Z_D(1)=$
$[0.9400 \quad 0.0600 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(2)=$
$[0.8600 \quad 0.1400 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(3)=$
$[0.5300 \quad 0.4500 \quad 0.0200 \quad 0.0000 \quad 0.0000]$,
$Z_D(4)=$
$[0.5800 \quad 0.4200 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(5)=$
$[0.6300 \quad 0.3700 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(6)=$
$[0.3000 \quad 0.6500 \quad 0.0500 \quad 0.0000 \quad 0.0000]$,
$Z_D(7)=$
$[0.8400 \quad 0.1600 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(8)=$
$[0.6700 \quad 0.3300 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(9)=$
$[0.8000 \quad 0.2000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_D(10)=$
$[0.7100 \quad 0.2900 \quad 0.0000 \quad 0.0000 \quad 0.0000]$.

Fuzzy comprehensive evaluation vectors for 10 teachers under the second-level factors *e1,e2,e3,e4 are*:

$Z_E(1)=$
$[0.8200 \quad 0.1800 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(2)=$
$[0.1500 \quad 0.8500 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(3)=$
$[0.7400 \quad 0.2600 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(4)=$
$[0.5000 \quad 0.5000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(5)=$
$[0.8800 \quad 0.1200 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(6)=$
$[0.8500 \quad 0.1500 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(7)=$
$[0.6600 \quad 0.3400 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(8)=$
$[0.5200 \quad 0.4500 \quad 0.0300 \quad 0.0000 \quad 0.0000]$,
$Z_E(9)=$
$[0.4000 \quad 0.6000 \quad 0.0000 \quad 0.0000 \quad 0.0000]$,
$Z_E(10)=$
$[0.5200 \quad 0.4800 \quad 0.0000 \quad 0.0000 \quad 0.0000]$.

The first-level fuzzy comprehensive evaluation matrix is constructed for the first teacher:

$$R_1 = \begin{bmatrix} Z_A(1) \\ Z_B(1) \\ Z_C(1) \\ Z_D(1) \\ Z_E(1) \end{bmatrix} = \begin{bmatrix} 0.7800 & 0.2200 & 0 & 0 & 0 \\ 0.5500 & 0.4200 & 0.0300 & 0 & 0 \\ 0.3000 & 0.6300 & 0.0700 & 0 & 0 \\ 0.9400 & 0.0600 & 0 & 0 & 0 \\ 0.8200 & 0.1800 & 0 & 0 & 0 \end{bmatrix}$$

The first-level fuzzy comprehensive evaluation vector of the first teacher is calculated, and

$$Z_1 = w \bullet R_1 = [0.287, 0.155, 0.260, 0.135, 0.163].$$

In the same way, the first-level fuzzy comprehensive evaluation vectors for the 2nd to the 10th teachers are:

$$Z_2 = w \bullet R_2 = [0.267, 0.175, 0.220, 0.195, 0.163],$$

$$Z_3 = w \bullet R_3 = [0.187, 0.185, 0.150, 0.145, 0.333],$$

$$Z_4 = w \bullet R_4 = [0.197, 0.140, 0.265, 0.130, 0.268],$$

$$Z_5 = w \bullet R_5 = [0.210, 0.155, 0.160, 0.135, 0.263],$$

$$Z_6 = w \bullet R_6 = [0.127, 0.285, 0.150, 0.165, 0.273],$$

$$Z_7 = w \bullet R_7 = [0.187, 0.175, 0.240, 0.145, 0.253],$$

$$Z_8 = w \bullet R_8 = [0.130, 0.257, 0.265, 0.175, 0.183],$$

$$Z_9 = w \bullet R_9 = [0.207, 0.185, 0.280, 0.168, 0.160],$$

$$Z_{10} = w \bullet R_{10} = [0.200, 0.158, 0.170, 0.185, 0.263].$$

3.6 Determining the evaluation results

According to the first-level fuzzy comprehensive evaluation vectors of 10 teachers, the comprehensive evaluation value of each teacher is calculated by using the weighted average principle, and the evaluation vector is used as the weighted sum of the weight and the corresponding rank of each component, that is,

$$N_i = \frac{1}{\sum_{k=1}^{5} Z_i^t(k)} \sum_{k=1}^{5} Z_i^t(k) \cdot k, i = 1, 2, \cdots, 10.$$

By taking $t = 1$, the comprehensive evaluation value of the first teacher is calculated as $N_1 = 0.890$. In the same way, the comprehensive evaluation values $N_2, \cdots, N_{10}$ of the remaining 9 teachers can be obtained. Eventually, the teachers are ranked on the basis of the size of the comprehensive evaluation values, and the specific results are shown in Table 9.

TABLE 9 RESULTS OF TEACHING EVALUATION FOR TEACHERS

Teachers' serial numbers	Comprehensive evaluation values	Rankings
1	0.890	3
2	0.785	8
3	0.749	9
4	0.833	6
5	0.843	5
6	0.824	7
7	0.740	10
8	0.867	4
9	0.899	2
10	0.915	1

The fuzzy comprehensive evaluation method is used to evaluate the classroom teaching of 10 teachers. Teacher 10 ranks first and teacher 7 ranks tenth. According to the data in table 2, the score data of teacher 7 and teacher 10 are visualized, as shown in Figure 1.

Through the analysis and comparison of Figure 1, it is found that teacher 10 has better performance than teacher 7 in

*a*1 Language communication skills, standardized Mandarin;
*b*1 Skillful ability to deliver lectures;
*b*2 Highlighting of key points and difficulties, proficient, thorough and clear lectures;
*b*3 Compliance with the syllabus, appropriate depth and breadth, and moderate progress;
*b*4 Ability to update knowledge;
*c*2 Ability to interact in the classroom;
*c*3 Ability to stimulate thinking;
*d*3 Careful preparation before class;
*e*2 Improvement of students' ability to learn and solve practical problems;

and other aspects. This demonstrates that Teacher 10's ranking ahead of Teacher 7 in the teaching evaluation is in line with reality, which also verifies the reasonableness of the evaluation results in this paper.

IV. Analysis and Generalization of the Results

The teaching quality evaluation of teachers is a complex and significant teaching work, which directly affects the learning effect and future development of students. By utilizing the fuzzy comprehensive evaluation method to analyze the teaching quality of 10 teachers, this paper overcomes the subjective arbitrariness of the traditional classroom teaching evaluation. By taking into account the multiple dimensions of teachers' teaching and their different focuses, it comprehensively evaluates the quality of each teacher's teaching, and makes the evaluation results more scientific and rational. In the meantime, the results of evaluation feedback provide a robust guarantee of the education and teaching quality.

The fuzzy comprehensive evaluation method has significant advantages in the evaluation of multi-level and multi-dimensional problems. On the one hand, it overcomes the error caused by uncertain factors in the evaluation process. On the other hand, its calculation process is easy to implement through MATLAB software, and easy to be mastered by teaching evaluation personnel, thus it solves the problem of "computational difficulty" when teachers or teaching management departments conduct multi-level and multi-dimensional classroom teaching quality evaluation.

The teaching ability of teachers is dynamic. On the basis of this study, combined with the theory of differential equations (see [9-12]), the long-term evaluation of teachers' teaching ability is made, which is the research direction of our future work.

TABLE 1 EVALUATION FACTORS

First-level factors	Second-level factors
A. Teaching literacy	*a1.* Language communication skills, standardized Mandarin *a2.* Blackboard writing and drawing ability (blackboard writing neat and standardized) *a3.* Level of knowledge, specialized basic knowledge and scientific research *a4.* Use of modern teaching methods
B. Teaching contents	*b1.* Proficiency in lecturing *b2.* Highly focused, skillful, thorough and clear lectures *b3.* Conformity with the syllabus, appropriate depth and breadth, and moderate pace of progression *b4.* Knowledge updating capacity
C. Teaching methods	*c1.* Teaching according to ability *c2.* Capacity for classroom interaction *c3.* Enlightened thinking skills *c4.* Ability to summarize
D. Teaching attitude	*d1.* Enthusiasm and positive energy in teaching *d2.* Caring, patient and concerned about student development *d3.* Careful preparation before class *d4.* Timely correction of homework and tutoring of assignments
E. Teaching effects	*e1.* Engage students' interest and deepen their cognition *e2.* Improvement of students' learning and practical problem-solving skills *e3.* Promote the cultivation of students' innovative ability and the improvement of their quality *e4.* Understanding and mastery of the content, with a solid "three fundamentals"

TABLE 2 RATING SCALE FOR TEACHERS' SECOND-LEVEL FACTORS

Scores		Teachers' serial numbers									
		1	2	3	4	5	6	7	8	9	10
Second level factors	*a1*	8.5	9.0	9.4	9.7	8.9	8.6	7.8	9.1	8.5	9.0
	a2	8.7	8.6	8.5	8.6	8.6	8.9	9.0	9.0	9.0	9.0
	a3	8.9	8.9	8.6	8.7	7.9	9.0	9.7	9.5	9.6	9.5
	a4	9.8	7.8	8.8	8.9	8.9	8.5	9.6	9.5	9.4	9.1
	b1	8.5	8.0	8.0	8.6	8.5	7.9	8.6	8.6	8.0	10
	b2	10	9.0	8.3	10	7.9	8.3	7.9	8.3	8.0	9.0
	b3	9.0	9.0	8.3	8.1	10	8.6	9.9	7.9	8.1	10
	b4	7.9	7.9	8.0	9.5	9.9	10	8.0	9.5	7.9	9.0
	c1	9.8	8.2	9.5	7.9	7.8	8.1	9.5	9.5	9.8	8.3
	c2	9.0	9.0	8.6	8.6	9.0	10	9.0	10	9.0	10
	c3	7.9	9.8	10	9.8	10	7.8	8.1	8.1	8.3	9.0
	c4	7.9	8.6	8.6	8.2	8.3	9.0	9.0	8.6	8.1	8.6
	d1	9.6	9.0	8.1	9.6	8.7	7.9	9.0	8.7	9.0	8.7
	d2	9.0	8.7	7.9	8.5	8.5	7.9	8.7	8.1	8.3	8.6
	d3	8.7	8.6	9.5	8.6	8.1	8.0	8.5	8.7	8.7	9.0
	d4	9.0	10	9.0	8.2	9.8	9.5	9.8	9.5	9.8	8.6
	e1	8.7	8.5	9.5	8.0	9.0	9.6	9.0	8.1	8.5	8.5
	e2	8.7	8.0	10	10	8.7	8.7	8.4	9.5	8.0	8.6
	e3	10	8.5	8.5	8.1	9.0	9.0	8.0	9.6	8.0	8.0
	e4	9.5	8.0	8.3	8.3	9.6	8.9	9.5	7.9	9.6	8.6

TABLE 4 RATING SCALE FOR TEACHERS' FIRST-LEVEL FACTORS

Scores		Teachers' serial numbers									
		1	2	3	4	5	6	7	8	9	10
First level factors	Teaching literacy	35.9	34.3	35.3	35.9	34.3	35.0	36.1	37.1	36.5	36.6
	Teaching contents	35.4	33.9	32.6	36.2	36.3	34.8	34.4	34.3	32.0	38.0
	Teaching methods	34.6	35.6	36.7	34.5	35.1	34.9	35.6	36.2	35.2	35.9
	Teaching attitude	36.3	36.3	34.5	34.9	35.1	33.3	36	35	35.8	34.9
	Teaching effects	36.9	33	36.3	34.4	36.3	36.2	34.9	35.1	34.1	33.7

TABLE 5 CHARACTERISTIC WEIGHTS

Characteristic weights		Teachers' serial numbers									
		1	2	3	4	5	6	7	8	9	10
First level factors	Teaching literacy	0.09	0.14	0.08	0.04	0.14	0.11	0.02	0.16	0.10	0.12
	Teaching contents	0.08	0.10	0.04	0.10	0.12	0.07	0.06	0.16	0.18	0.09
	Teaching methods	0.10	0.11	0.13	0.10	0.10	0.11	0.09	0.06	0.11	0.09
	Teaching attitude	0.05	0.10	0.12	0.09	0.10	0.12	0.16	0.10	0.09	0.07
	Teaching effects	0.10	0.13	0.07	0.13	0.05	0.12	0.12	0.08	0.10	0.10

TABLE 6 INFORMATION ENTROPY TABLE FOR FIRST-LEVEL FACTORS

First-level factors	Teaching literacy	Teaching contents	Teaching methods	Teaching attitude	Teaching effects
Information entropy (e_j)	0.80	0.90	0.80	0.80	0.70

TABLE 7 COEFFICIENTS OF VARIATION FOR FIRST-LEVEL FACTORS

First-level factors	Teaching literacy	Teaching contents	Teaching methods	Teaching attitude	Teaching effects
Coefficient of variation (g_j)	0.20	0.10	0.20	0.20	0.30

TABLE 8 WEIGHTS OF FIRST-LEVEL FACTORS

First-level factors	Teaching literacy	Teaching contents	Teaching methods	Teaching attitude	Teaching effects
Weight (w_j)	0.20	0.10	0.20	0.20	0.30

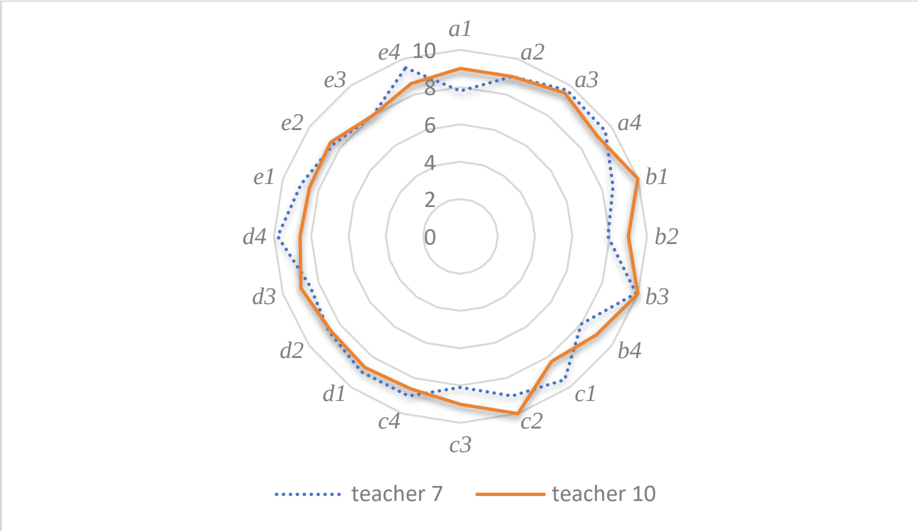

Fig 1 Comparison of teacher 7 and teacher 10's teaching evaluation scores

References

[1] L. Lai, X. He, X. Chen, "Evaluation of online teaching service quality based on fuzzy comprehensive evaluation method", *Journal of Higher Learning Research*, vol. 8, no. 9, pp60-64, 2022.

[2] Y. Luo, "A model for teaching evaluation in open math classes", *Mathematics Learning and Research*, no. 26, pp152-153, 2020.

[3] X. Shao, "Fuzzy comprehensive evaluation of health level of Shanghai residents based on entropy method", *Journal of Economic Research*, no. 36, pp92-94, 2022.

[4] H. Li, "Research and application of weighting algorithm of used car renewal rate based on entropy method", *Chongqing University of Technology*, 2019.

[5] Z. Ye, "Research and application of fuzzy comprehensive evaluation method based on AHP", *South China University of Technology*, 2018.

[6] F. Zhang, Y. Hu, "Application of fuzzy comprehensive judgment and hierarchical analysis method in teaching evaluation of colleges and universities", *Journal of Changchun Normal University*, no. 6, pp10-13, 2006.

[7] Z. Pei, X. Zhu, C. Gong, "Weight assignment of DRG index evaluation system based on the combination of hierarchical analysis and entropy method", *China Hospital Management*, vol. 40, no. 11, pp69-72, 2020.

[8] Y. Zhang, X. Wei, Z. Chen. Research on dual subject teaching satisfaction under the mixed reform of higher vocational secondary colleges--Based on entropy weight and fuzzy comprehensive evaluation model. *Southern Agricultural Machinery*, vol. 53, no. 5, pp190-194, 2022.

[9] Lili Wang, and Pingli Xie, "Almost periodic solutions in shifts delta(+/-) of nonlinear dynamic equations with impulses on time scales," *IAENG International Journal of Applied Mathematics*, vol. 54, no. 2, pp249-254, 2024.

[10] Meng Hu, and Lili Wang, "Almost periodicity in shifts delta(+/-) on time scales and its application to Hopfield neural networks," *Engineering Letters*, vol. 29, no.3, pp864-872, 2021.

[11] Lili Wang, Pingli Xie, and Yuwei Wang, "Almost and pseudo almost periodic solutions in shifts delta(+/-) for nonlinear dynamic equations on time scales with applications," *IAENG International Journal of Applied Mathematics*, vol. 53, no. 3, pp1094-1099, 2023.

[12] Lili Wang, and Haiyan Lv, "Extinction in an impulsive nonautonomous Lotka-Volterra competitive system with discrete delay and infinite delay," *IAENG International Journal of Applied Mathematics*, vol. 50, no.2, pp373-378, 2020.