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Column Generation for Transmission Switching of Electricity Networks with Unit Commitment

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***Abstract*—This paper presents the problem of finding the minimum cost dispatch and commitment of power generation units in a transmission network with *active switching*. We use the term *active switching* to denote the use of switches to optimize network topology in an operational context. We propose a Dantzig-Wolfe reformulation and a novel column generation framework to solve the problem efficiently. Preliminary results are presented for the IEEE-118 bus network with 19 generator units. Active switching is shown to reduce total cost by up to 15 % for a particular 24-hour period. Furthermore, the need for generator startups is reduced by 1. Instances with limited switching, some of which are intractable for commercial solvers, are shown to solve to optimality in reasonable time.**

I. Introduction

In meshed electricity transmission networks Kirchhoff's laws constrain the flow on each line in a cycle. In the DC-load flow approximation, the power flow on any line must be proportional to the voltage phase angle difference for the two end nodes. For power flow in a tree network, the power flows are uniquely determined by flow conservation at the nodes. For any feasible flow the voltage phase angles are then uniquely determined up to an additive constant, and so they do not affect the economic dispatch.

When the network contains cycles, the voltage phase angles affect the dispatch. This is important in practice, since most electricity transmission networks are designed as meshed networks (with cycles) for security reasons, so that if any line fails, the power can still flow from source to destination by alternative paths. In a meshed network, voltage phase angles become important, since they result in additional constraints on the line flows. In particular, for each cycle in a network the sum of voltage angle differences (with respect to the direction) around the cycle must equal zero. Hence, each cycle in the network gives rise to one additional constraint on the line flows. This leads to a paradox (see e.g. [1]) in which adding a new line to a transmission network might increase the cost of supplying electricity, even if the cost of the line itself is zero.

Based on these observations, it is easy to see that it may be beneficial in mesh networks to take some lines out of operation — to either decrease system cost or increase reliability [2], [3]. The process of taking out lines and bringing them back in is done by opening (respectively closing) a switch at the end of the line and is referred to as *switching*.

Recent interest in renewable intermittent energy sources and the call for intelligent transmission networks or smart grids have spurred a renewed interest in switching problems. Fisher et. al. presents in [4] the problem of optimal switching of transmission elements in an electricity transmission network to minimize the delivered cost of energy. They propose a mixed-integer program to solve the DC-approximated load-flow with switching decisions in a single time period. They note that the problem is NP-hard. Results are provided for a 118-node network with 186 transmission lines. Hedman et. al. [5] extends the model to consider reliability of the network. Reliability constraints are added to the problem to ensure that any line failure will not lead to an infeasible dispatch of generation. They note that in some cases adding reliability constraints increases the value of switching.

In [6] Hedman et. al. discuss a decomposition algorithm to solve the security constrained transmission switching problem with unit commitment decisions made heuristically over 24 time periods. The master problem handles unit commitment decisions over the planning horizon given a fixed switching configuration of the lines, while sub-problems — one for each time period — optimizes the switching configuration given a fixed unit commitment plan. It is noted that adding transmission switching may yield a cheaper unit commitment plan with fewer start-ups than what could be achieved without switching. Khodaei and Shahidehpour [7] propose a Benders decomposition of the security constrained unit commitment and transmission switching problem.

In this paper, we assume that a technology is available that makes it possible to switch lines instantaneously. That is, a line may be switched automatically from one moment to the next without delay. In this case, switching out lines will (in theory) not affect system security (disregarding failures on switching equipment), since all lines may be switched back in immediately, in case of any failure in the system.

We propose a Dantzig-Wolfe reformulation and column generation framework for the transmission switching and unit commitment problem. In this approach, each subproblem generates a feasible switching and unit commitment plan for a single time period, while the master problem makes a selection from the set of generated plans so as to minimize the total cost of generation. In this paper we disregard security constraints and other special constraints such as minimum up- and down time, ramp rate, and reserve constraints. Results show that employing active switching may reduce generation cost by up to 15 % and save generator startup costs. This is in line with results obtained in [6].

The paper is laid out as follows. Section II describes a deterministic minimum cost dispatch DC-approximated load flow model with transmission switching for a single time period. In section III we look at the multi-period problem with start-up costs and propose a Dantzig-Wolfe reformulation and column generation framework for finding (near-) optimal solutions. In section IV we present some

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results for the IEEE 118-bus network. Section V concludes the paper and gives some directions for future research.

II. Optimal Dispatch with Transmission Switching and Unit Commitment

Consider an electricity transmission network, where N denotes the set of nodes (or buses) and E denotes the set of transmission lines (and transformers) connecting the nodes. Let $\mathcal{T}(i)$ denote the set of lines incident with node i where i is the head of the incident lines, and let $\mathcal{F}(i)$ denote the set of lines incident with node i, where i is the tail of the incident lines. So a line in $\mathcal{F}(i) \cap \mathcal{T}(j)$ is oriented *from i to j*.

Many transmission systems consist of alternating current circuits, interlinked by high voltage direct current links. We shall ignore these in this paper, and assume that all lines carry alternating current. The methodologies can easily be adapted to treat high voltage direct current lines as special cases.

Let G be the set of all generating units, that may offer electricity to the market. Furthermore, let $G(i)$ be the set of generating units located in (and supplying electricity to) node i. Each generating unit g offers a price c_g and a quantity u_g of energy to be generated. If the offer is accepted, unit g will deliver the quantity q_g to the market (assuming that a generator will never offer more than its generating capacity).

At each node i the demand d_i must be met. Load shedding at node i may be modelled by introducing a dummy generator g' offering the quantities $q_{g'}$ at the corresponding (sufficiently high) price $c_{g'}$ such that $q_{g'} \le \max(0, d_i) = u^q_{g'}$.

Each transmission line $e \in E$ is characterized by its reactance R_e and thermal capacity u_e. The flow on line e is denoted x_e, which can be negative to model power flows in the direction opposite to the orientation of e. All lines are assumed to be switchable and may be taken out of operation in any given period of time. For each line $e \in E$, $z_e = 0$ denotes that the line has been switched out (opened), while $z_e = 1$ denotes that the switch is closed.

The minimum cost dispatch problem for a single period assuming no start-up cost may now be formulated as,

$$\min \sum_{g \in G} c_g q_g \tag{1}$$

s.t.

$$l_g z_g \le q_g \le u^q_g z_g, \qquad \forall g \in G \tag{2}$$

$$d_i + \sum_{e \in \mathcal{F}(i)} x_e = \sum_{g \in G(i)} q_g + \sum_{e \in \mathcal{T}(i)} x_e, \quad \forall i \in N \tag{3}$$

$$-u_e z_e \le x_e \le u_e z_e, \qquad \forall e \in E \tag{4}$$

$$z_e = 1 \;\Rightarrow\; R_e x_e = \sum_{i: e \in \mathcal{T}(i)} \theta_i - \sum_{i: e \in \mathcal{F}(i)} \theta_i, \quad \forall e \in E \tag{5}$$

$$0 \le q_g, \qquad \forall g \in G \tag{6}$$

$$z_g, z_e \in \{0, 1\}, \qquad \forall g \in G, e \in E \tag{7}$$

The objective (1) minimizes the total generation costs respecting generation capacities and minimum generation on committed units (2), flow conservation (3), and thermal line capacity (4). For lines that are not switched out Kirchhoff's voltage law must be respected (5). Generation quantities are non-negative (6), and switching and unit commitment decisions are binary (7).

Since (2) - (7) is NP-hard [4], we may — for computational reasons — limit the number of lines to be switched simultaneously to at most k:

$$\sum_{e \in E} (1 - z_e) \le k, \tag{8}$$

Note, that constraints (5) may be linearized using the big-M notation,

$$-M(1 - z_e) \le R_e x_e - \sum_{i: e \in \mathcal{T}(i)} \theta_i + \sum_{i: e \in \mathcal{F}(i)} \theta_i, \; \forall e \in E \tag{9}$$

$$M(1 - z_e) \ge R_e x_e - \sum_{i: e \in \mathcal{T}(i)} \theta_i + \sum_{i: e \in \mathcal{F}(i)} \theta_i, \; \forall e \in E \tag{10}$$

where M is some sufficiently large number.

III. Multi-Period Formulation and Dantzig-Wolfe Reformulation

We now consider the problem of finding a minimum cost dispatch of generation and commitment of generator units in an electricity transmission network with active switching over several timeperiods.

Consider the discretized planning horizon Ω as a set of discrete time periods. For each period $\omega \in \Omega$, let $\mathcal{Q}(\omega)$ denote the set of feasible operational decisions (q, x, θ, z) satisfying constraints (2) - (8) . The multi-period model may now be formulated as,

$$\min \sum_{\omega \in \Omega} \left(f^\top y(\omega) + c(\omega)^\top q(\omega) \right) \tag{11}$$

s.t.

$$z_g(\omega) - z_g(\omega - 1) \;\le\; y_g(\omega), \quad \forall g \in G, \omega \in \Omega \tag{12}$$

$$(q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega), \quad \forall \omega \in \Omega \tag{13}$$

$$y(\omega) \in \{0, 1\}^{|G|}, \; \forall \omega \in \Omega \tag{14}$$

The objective (11) minimizes the hourly fixed and operational cost, while (12) ensures that fixed unit commitment cost is incurred if the unit is on in the current time period ω and off in the previous time period $\omega - 1$. We assume, that the planning period is cyclic so that for the first element ω' of Ω, $\omega' - 1$ refer to the last element of Ω.

We propose a Dantzig-Wolfe reformulation of the multi-period model following the approach in [8]. First, let the binary vector $z(\omega)$ define a feasible switching and unit commitment plan (FSUP) for time period ω, if and only if, there exists $q(\omega), x(\omega), \theta(\omega)$ such that $(q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega)$. The idea is to decompose the multi-period problem into a master problem and a number of subproblems — one for each time period. Each of the subproblems generate feasible switching and unit commitment plans for the corresponding time period, while the master problem chooses among the generated FSUP's and determines the optimal unit commitment strategy.

Now, let $Z(\omega) = \{\hat{z}^j(\omega) | j \in J(\omega)\}$ be the set of FSUP's in time period ω, where $J(\omega)$ is the index set for $Z(\omega)$. We can now write any element in $Z(\omega)$ as

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$$\begin{aligned} z(\omega) &= \sum_{j \in J(\omega)} \varphi^j(\omega) \hat{z}^j(\omega), \\ \sum_{j \in J(\omega)} \varphi^j(\omega) &= 1, \quad \varphi^j(\omega) \in \{0,1\}, \ \forall j \in J(\omega) \end{aligned}$$

Assume that for each feasible switching and unit commitment plan $\hat{z}^j(\omega)$ the corresponding optimal dispatch of generation and load shedding is given by $\hat{q}^j(\omega)$. The master problem can now be written in terms of $\hat{z}$ and $\hat{q}$, that is

$$\min \quad \sum_{\omega \in \Omega} \left(f^\top y(\omega) + \sum_{j \in J(\omega)} c(\omega)^\top \hat{q}^j(\omega) \varphi^j(\omega) \right) \quad (15)$$

s.t.

$$\sum_{j \in J(\omega)} \varphi^j(\omega) = 1, \qquad [\mu(\omega)] \ \forall \omega \in \Omega \quad (16)$$

$$\sum_{j \in J(\omega)} \hat{z}^j(\omega)\varphi^j(\omega) \le y(\omega) + \sum_{j \in J(\omega-1)} \hat{z}^j(\omega-1)\varphi^j(\omega-1), \qquad [\pi(\omega)] \ \forall \omega \in \Omega \quad (17)$$

$$\varphi^j(\omega) \in \{0,1\}, \qquad \forall j \in J(\omega), \omega \in \Omega \quad (18)$$

$$y(\omega) \in \{0,1\}^{|G|}, \quad \forall \omega \in \Omega \quad (19)$$

where $\mu(\omega)$ and $\pi(\omega)$ denote the dual prices for constraints (16) respectively (17).

It is convenient to consider only a subset $Z'(\omega) \subseteq Z(\omega)$ of feasible switching and unit commitment plans for each time period ω in the master problem. We define this restricted master problem (RMP) by (15) - (19) with $J(\omega)$ replaced by $J'(\omega)$ the index set of $Z'(\omega)$. A column generation algorithm is applied to dynamically add FSUP's to the linear relaxation of the master problem. The algorithm is initialized by letting $Z'(\omega) = \{z^0(\omega)\} = \{\mathbf{1}\}$, for all time periods $\omega \in \Omega$. That is, initially no line may be switched and all units are comitted in all time periods. The corresponding operational costs $c^\top(\omega)\hat{q}^0(\omega)$ can easily be found by solving a linear program for each time period. In each iteration of the algorithm the linear relaxation (RMP-LP) of RMP is solved yielding the dual prices μ, π. A new column $(c^\top(\omega)\hat{q}^j(\omega), 1, \hat{z}^j(\omega))$ may improve the solution of RMP-LP if and only if the associated reduced cost $\bar{c}(\omega) = c^\top(\omega)\hat{q}^j(\omega) - (\hat{\pi}^\top(\omega) - \hat{\pi}^\top(\omega+1))\hat{z}^j(\omega) - \hat{\mu}(\omega)$ is negative.

A column for time period ω may be constructed by solving the subproblem,

$$\min \ c^\top(\omega)q(\omega) - (\hat{\pi}^\top(\omega) - \hat{\pi}^\top(\omega+1))z(\omega) - \hat{\mu}(\omega) \quad (20)$$

$$\text{s.t.} \qquad (q(\omega), x(\omega), \theta(\omega), z(\omega)) \in \mathcal{Q}(\omega) \quad (21)$$

where $\hat{\pi}(\omega)$ and $\hat{\mu}(\omega)$ are the dual prices returned from RMP-LP. Note, that each subproblem is in fact a network design problem with single commodity flow and Kirchhoff's voltage requirements (5) — see eg. [9].

Any feasible solution $(\hat{q}(\omega), \hat{x}(\omega), \hat{\theta}(\omega), \hat{z}(\omega)) \in \mathcal{Q}_\omega$ with negative objective function gives rise to a potential candidate column for RMP-LP. Hence, we do not rely on finding optimal solutions to the subproblems. Since our subproblems are NP-hard mixed integer programs (and potentially large for realistic size transmission networks) we may settle with suboptimal solutions in favour of generating more columns. In fact, it is not necessary to generate solutions to all the subproblems in each iteration, and hence we may postpone the generation of columns for subproblems, where a solution with negative reduced cost is not easily obtained. When all subproblems return solutions with non-negative reduced costs $\bar{c}$ we resolve all subproblems to optimality if necessary.

If no columns with negative reduced cost exist we have solved the relaxed master problem (MP-LP) to optimality. Furthermore, if the solution (φ^*, y^*) to MP-LP is integral, (φ^*, y^*) is an optimal solution to the master problem (15) - (19) and y^* is the optimal unit commitment strategy. Otherwise, we may resort to a branch-and-bound framework for finding optimal integral solutions or simply solve the integral RMP in the hope of finding a good feasible integer solution.

When the relaxed master problem MP-LP yields fractional solutions one may resort to branching to attain integral solutions. In this paper a crude one-level branching scheme, where we branch on one of the y-variables, is proposed to resolve fractionality of the relaxation. This does not guarantee optimal integral solutions in general, but yields feasible near-optimal solutions in practice. A branch-and-price scheme — in which one continue to branch on fractional variables until fractionality is resolved — may be employed to guarantee optimal integral solutions in general.

IV. Computational Results

In this section, we apply the column generation algorithm proposed above to the IEEE 118-bus network [10] with network data described in [11]. This network has 185 lines, 19 generator units, total peak load of 4519 MW, and a total thermal generator capacity of 5859 MW. We consider the demand data [10] for day 2 (winter, weekend) with 24 hourly time periods and assume a minimum generation level of 20 % of capacity and start-up cost of \$ 10 for each of the 19 generator units.

The decomposition and models are formulated using the AMPL modelling language and all master- and subproblems are solved with CPLEX 12.2. The relaxed master problems are solved using CPLEX barrier algorithm (without applying crossover at the end), while the subproblems are solved using the CPLEX standard branch-and-bound algorithm. Computational experiments are performed on a 2.26 GHz Core 2 Duo computer with 4 GB RAM.

The power dispatch problem with unit commitment is solved for different values of k, where $k = 0$ characterizes the instance without switching. Table I shows solutions and running times for the column generation and branch-and-bound algorithm for problem instances with 24 (hourly) time periods. For the branch-and-bound algorithm CPLEX 12.2 was applied with default parameters.

The column generation algorithm solved all instances with $k \le 4$ to optimality in the root node and hence no branching was needed. For $k = 8$ the algorithm terminated without proving optimality or even providing a lower bound. In general, integrality is not guaranteed and branching on fractional variables may be necessary to obtain integral optimal solutions. Only a small fraction of the total solve time used

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TABLE I
SOLUTION TIMES AND ABSOLUTE GAP TO THE OPTIMAL SOLUTION FOR PROBLEM INSTANCES WITH 24 TIME PERIODS FOR THE PROPOSED COLUMN GENERATION ALGORITHM AND STANDARD BRANCH-AND-BOUND (CPLEX). FOR THE COLUMN GENERATION ALGORITHM THE TIME USED IN THE SUBPROBLEMS, THE NUMBER OF ITERATIONS AND THE NUMBER OF COLUMNS ADDED IS ALSO SHOWN. INSTANCES MARKED BY $\star$ ARE SOLVED TO INTEGRALITY IN THE ROOT NODE, WHILE † DENOTES THAT OPTIMISATION WAS TERMINATED WITHOUT PROVING OPTIMALITY. FOR $k > 0$ THE BRANCH-AND-BOUND ALGORITHM WAS TERMINATED DUE TO LACK OF MEMORY. ‡ INDICATES THAT OPTIMALITY WAS NOT PROVEN.

Instance		Column generation					Branch-and-bound		Best known solution	
network	k	time (s)	time, sub (s)	abs. gap	ite.	col.	time (s)	abs. gap	value	start-ups
IEEE118	0	46.6	46.0	0	16 $\star$	84	8	0	28128.35	8
IEEE118	1	1996.1	1995.5	0	18 $\star$	80	-	1353.52	26580.22	6
IEEE118	2	5531.3	5531.0	0	7 $\star$	70	-	2467.23	25898.55	7
IEEE118	3	15681.4	15680.9	0	9 $\star$	69	-	2434.40	25215.01	7
IEEE118	4	29665.3	29664.8	0	13 $\star$	83	-	2779.21	24978.30	7
IEEE118	8	45553.0	45552.4	-	17 †	106	-	-	23884.08 ‡	7

by the column generation algorithm is spent solving the master problems. The majority of the time is spent in the subproblems. Future research should be directed at solving the subproblems efficiently.

Without switching ($k = 0$) the branch-and-bound algorithm proved more efficient than column generation. However, for $k > 0$ the branch-and-bound algorithm ran out of memory and the best feasible solution returned was considerably worse than the solution returned by the column generation algorithm.

In the situation without switching the total generation and unit commitment cost incurred is \$ 28128.35 and a total of 8 start-ups are required. When allowing switching of at most four switches in each time period the total cost is reduced to \$ 24978.3. For $k = 8$ the total cost is further reduced to \$ 23884.08 with 7 generator start-ups.

V. CONCLUSION

In this paper we consider the problem of determining an optimal dispatch and unit commitment of power generation in a transmission network with active switching. We propose a Dantzig-Wolfe reformulation of the multi-period formulation into a master problem handling start-ups over the entire planning horizon and a number of subproblems each of which generates feasible unit commitment and switching patterns for a single time period. A column generation approach is outlined to solve the Dantzig-Wolfe reformulation.

The effect of allowing active switching in a setting with start-up costs on generator units is evaluated on the IEEE-118 bus network. Computational results show that over a particular 24-hour period total cost is reduced by up to 15 % and the number of start-ups are reduced by 1.

Furthermore, the proposed column generation algorithm is shown to be significantly more efficient than solving the problem using CPLEX standard branch-and-bound with default options. However, due to the computational complexity of the subproblems the algorithm spends the majority of the time solving the subproblems. Hence, further research should be directed at providing stronger formulations and more efficient solution methods for the subproblems, in order to improve the overall efficiency of the algorithm.

The model in this paper disregards security constraints, ramp rate constraints, and other generation specific constraints. Security constraints have been noted by Hedman et. al. [6] to increase the computational complexity of the problem significantly. Future research should investigate the impact of such constraints on the running times of the column generation algorithm.

The algorithm employed in this paper is a first step showing proof of concept — and it may not always return optimal integer solutions. Integrality may be ensured by employing a general branch-and-price scheme, where we continue branching on fractional variables until an integer solution is obtained. Optimizing the algorithm design may speed up solution times as well as ensure optimal solutions in general.

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