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Simulations on Motion Control and Development of Biped Walking Robot

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Abstract—**The purpose of this research is to make the biped walking robot walk considering the theory of passive walking. Two motors were installed at the hip joints connecting both legs to torso link. The numerical calculations about walking motion were performed. The proportional (P) or proportional-differential (PD) control law was used in the calculations. The experimental robot has been being developed considering the calculated results.**

Index Terms—**Biped walking robot, Numerical calculation, Passive walking, Active walking, Control**

I. Introduction

WALKING efficiency of the biped walking robots is inferior to that of human beings and animals. It is important to improve walking efficiency of the robots. The passive walking robots[1] have been researched to improve walking efficiency.

The passive walking robots can efficiently walk on the slope by using their legs such as pendulums. However the passive walking robots can't walk on the horizontal ground because the actuators aren't installed on them. In this research, two motors were installed at the hip joints of the robot in order to load the control torques. Then the robot was able to walk on the horizontal ground in the numerical calculations. The comparisons of the attitude angle of the torso and the control torques were performed when the proportional or proportional-differential control was used. The experimental robot under development is presented.

II. Model of Walking Robot

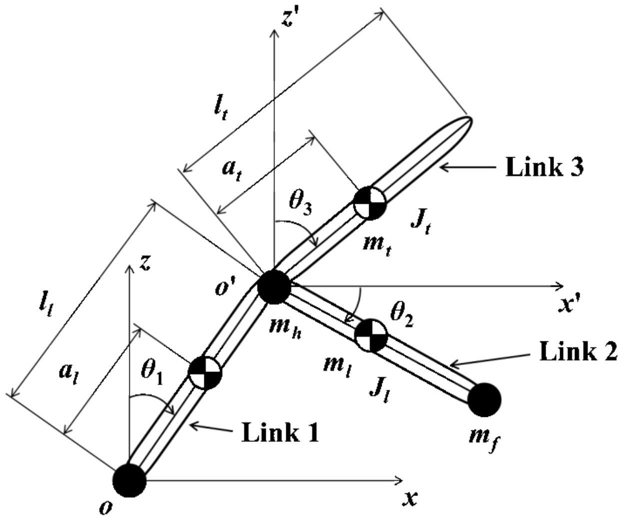
Figure 1 shows the analysis model of the walking robot. The robot is composed of a torso, a hip and two legs with feet. Figure 2 shows the experimental walking robot installing two motors at the hip joints. The τ_1 denotes the control torque between the stance-leg and the torso. The τ_2 also denotes the control torque between the swing-leg and the torso. The motion of the robot was constrained in the two dimensional plane. Then Scilab was used as a computer language.

A. Equation of Motion

The equation of motion is given by

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \left[\boldsymbol{C_c}\left(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}\right) + \boldsymbol{C_d}\right]\dot{\boldsymbol{\theta}} + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{B\tau} \quad (1)$$

where $\boldsymbol{\theta} = \{\theta_1 \quad \theta_2 \quad \theta_3\}^T$, $\boldsymbol{\tau} = \{\tau_1 \quad \tau_2\}^T$. The components of matrices $\boldsymbol{M}$, $\boldsymbol{C_c}$, $\boldsymbol{C_d}$ and $\boldsymbol{B}$ and vector $\boldsymbol{g}$ are shown in the Appendix. The matrix $\boldsymbol{C_d}$ means the viscosity damping, and its components were set at zero in these calculations.



Link 1: stance-leg, Link 2: swing-leg, Link 3: torso
m_l : mass of the leg, m_f : mass of the foot, m_h : mass of the hip, m_t : mass of the torso,
a_l : length from the foot to the gravity center of the leg,
a_t : length from the hip to the gravity center of the torso,
l_l : length of the leg, l_t : length of the torso,
J_l : inertia moment of the leg, J_t : inertia moment of the torso

Fig. 1. Analysis model of walking robot.

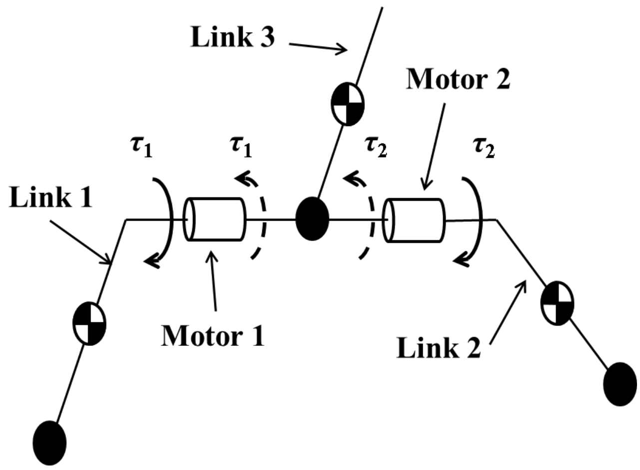


Fig. 2. Components of experimental walking robot.

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B. Way to Switch Legs

The condition of the touchdown of the swing-leg is given by

$$\theta_1 + \theta_2 = \pi/2. \tag{2}$$

The contact between the swing-leg and the ground when the swing-leg passes through the stance-leg was ignored. Then the impact at the touchdown was assumed to be inelastic. The stance-leg was assumed to leave from the ground at the moment of the touchdown.

The angular momentum is conserved before and after the touchdown for the whole robot about the leading contact point, the trailing leg about the hip and the torso about the hip[2]. Equation (3) is given by these conservation laws of angular momentum:

$$\boldsymbol{Q}^{+}(\boldsymbol{\theta}^{+})\dot{\boldsymbol{\theta}}^{+} = \boldsymbol{Q}^{-}(\boldsymbol{\theta}^{-})\dot{\boldsymbol{\theta}}^{-}. \tag{3}$$

The components of matrices $\boldsymbol{Q}^{+}$ and $\boldsymbol{Q}^{-}$ are shown in the Appendix. The each superscript "-" or "+" denote the state before or after the touchdown. The relation of the angles before and after the touchdown is given by

$$\boldsymbol{\theta}^{+} = \boldsymbol{R}\boldsymbol{\theta}^{-} + \boldsymbol{\theta_0}, \tag{4}$$

where

$$\boldsymbol{R} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

$$\boldsymbol{\theta_0} = \{-\pi/2 \quad \pi/2 \quad 0\}^T, \tag{6}$$

Equation (7) is given by (3):

$$\dot{\boldsymbol{\theta}}^{+} = \boldsymbol{Q}^{+}(\boldsymbol{\theta}^{+})^{-1}\boldsymbol{Q}^{-}(\boldsymbol{\theta}^{-})\dot{\boldsymbol{\theta}}^{-}, \tag{7}$$

The angular velocities of respective links after the touchdown were calculated by using (7).

III. MOTION CONTROL

A. Proportional Control

The control torques τ_1 and τ_2 loaded to the torso and the swing-leg are each given by (8) and (9) when the proportional control is used:

$$-\tau_1 - \tau_2 = k_3^p(\theta_{3d} - \theta_3), \tag{8}$$

$$\tau_2 = k_2^p(\theta_{12d} - \theta_{12}), \tag{9}$$

where

$$\theta_{12} = \theta_1 + \theta_2, \tag{10}$$

$$\theta_{12d} = \pi/2. \tag{11}$$

The k_2^p and k_3^p denotes the control gains. The control torque τ_1 is given by (8) and (9):

$$\tau_1 = -k_2^p(\theta_{12d} - \theta_{12}) - k_3^p(\theta_{3d} - \theta_3). \tag{12}$$

B. Proportional-Differential Control

The control torques loaded to the torso and the swing-leg are each given by (13) and (9) when the proportional-differential control is used:

$$-\tau_1 - \tau_2 = k_3^p(\theta_{3d} - \theta_3) - k_3^d\dot{\theta}_3. \tag{13}$$

Here the k_3^d denotes the control gain. The τ_1 is given by (9) and (13):

$$\tau_1 = -k_2^p(\theta_{12d} - \theta_{12}) - k_3^p(\theta_{3d} - \theta_3) + k_3^d\dot{\theta}_3. \tag{14}$$

The block diagram of an active control for the calculations is shown in Figure 3 when the proportional-differential control is used.

IV. REASONABLE GAINS USING P AND PD CONTROLS

A. Physical Parameters of Analysis Model

The parameters of the lengths, the masses and the inertia moments of the links used in the calculations are shown in Table 1.

B. Proportional Control

The reasonable control gains are suggested when the proportional control is used. The number of walking of the robot was examined when the desired attitude angle θ_{3d} of the torso was changed from 0[deg] to 45[deg] every 5[deg]. We thought here the robot could walk sufficiently when the number of walking was more than 100 steps. The reasonable control gains mean the minimum control gains in the case that the robot can walk 100 steps or more. The control gain k_2^p was changed from 1.0[N·m/rad] to 2.5[N·m/rad] every 0.1[N·m/rad]. Then the control gain k_3^p was changed from 1.0[N·m/rad] to 15.0[N·m/rad] every 1.0[N·m/rad]. The relations between the control gains and the number of walking are shown in Figure 4. It was found that the larger control gains are needed when θ_{3d} becomes larger.

Table 1. Physical parameters of analysis model.

m_l [kg]	3.22×10^{-2}	a_t [m]	7.50×10^{-2}
m_f [kg]	3.43×10^{-3}	l_l [m]	3.00×10^{-1}
m_h [kg]	3.00×10^{-2}	l_t [m]	1.50×10^{-1}
m_t [kg]	3.39×10^{-1}	J_l [kg·m^2]	2.43×10^{-4}
a_l [m]	1.50×10^{-1}	J_t [kg·m^2]	6.47×10^{-4}

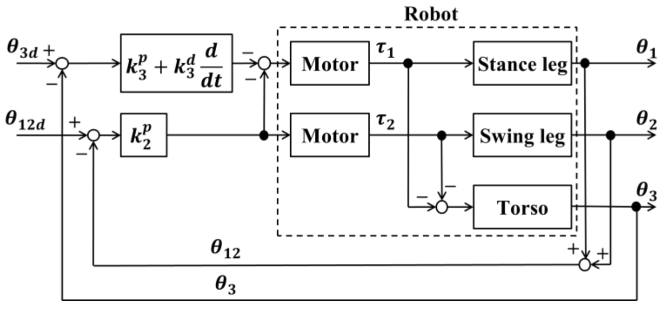

Fig. 3. Block diagram of an active control for calculations.

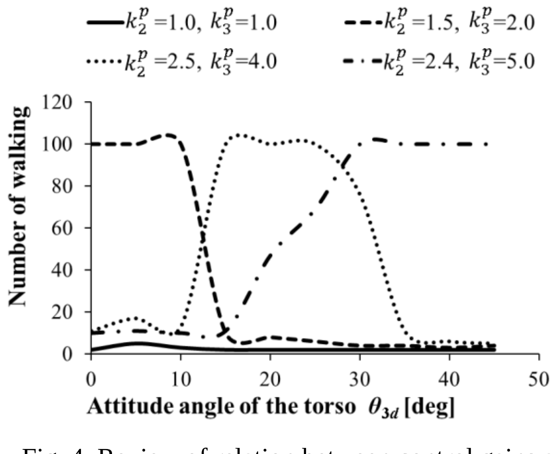

Fig. 4. Review of relation between control gains and the number of walking.

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C. Proportional-Differential Control

The reasonable control gain is suggested when the proportional-differential control is used. The desired attitude angle θ_{3d} of the torso was set at 5[deg]. The values of k_2^p=1.5[N·m/rad] and k_3^p=2.0[N·m/rad] were used as the proportional control gains. The control gain k_3^d was examined about 0.01, 0.1 and 1.0[N·m·s/rad]. As this result, it was found that k_3^d=0.1[N·m·s/rad] was the reasonable value.

V. Comparison between P and PD Controls

A. Attitude Angle of Torso

The time history response of the attitude angle θ_3 of the torso is shown in Figure 5. It was found that θ_3 was stabilized by using the proportional-differential control.

B. Control Torque

The time history responses of the control torques τ_1 and τ_2 are each shown in Figure 6 and Figure 7. It was found that the difference between magnitudes of τ_1 and τ_2 was not so large. The walking motion calculated using the proportional-differential control is shown in Figure 8.

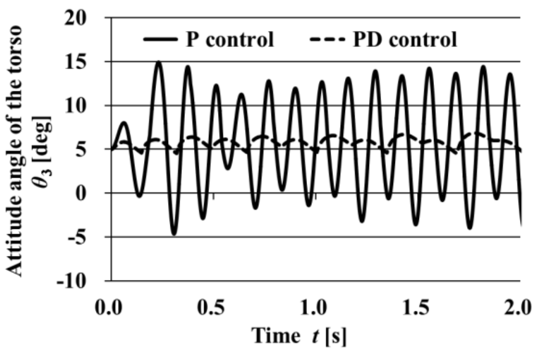

Fig. 5. Comparison of θ_3 between P and PD controls.

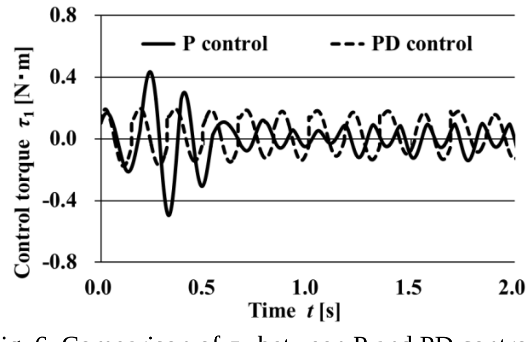

Fig. 6. Comparison of τ_1 between P and PD controls.

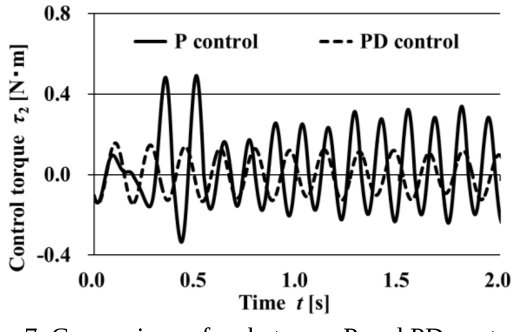

Fig. 7. Comparison of τ_2 between P and PD controls.

VI. Development of Experimental Robot

The experimental robot has been being developed considering the calculated results. The photo of the robot under development is shown in Figure 9. The system configuration of the robot is shown in Figure 10. The block diagram of an active control for the experimental robot is shown in Figure 11 when the proportional-differential control is used. The lengths of the leg and the torso are 310[mm] and 140[mm], respectively.

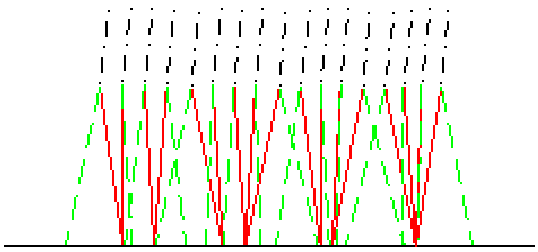

Fig. 8. Walking motion using PD control.

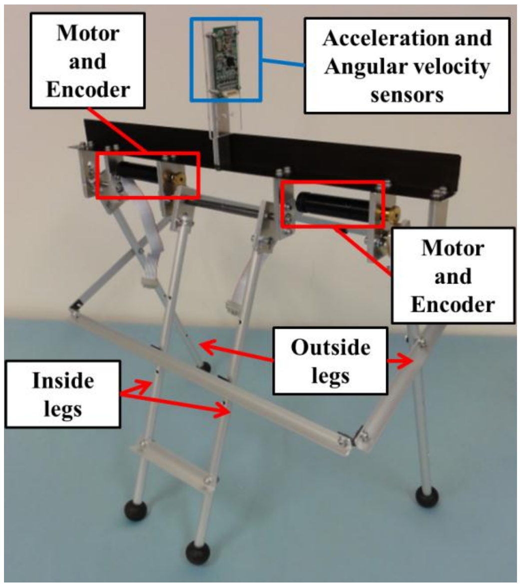

Fig. 9. Experimental robot under development.

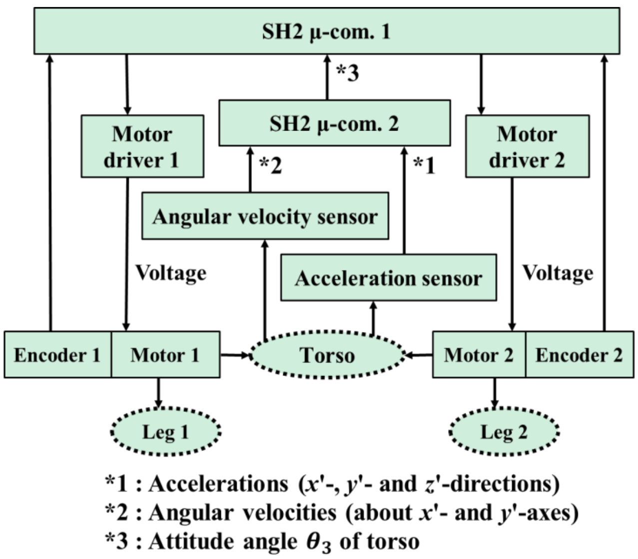

Fig. 10. System configuration of experimental robot.

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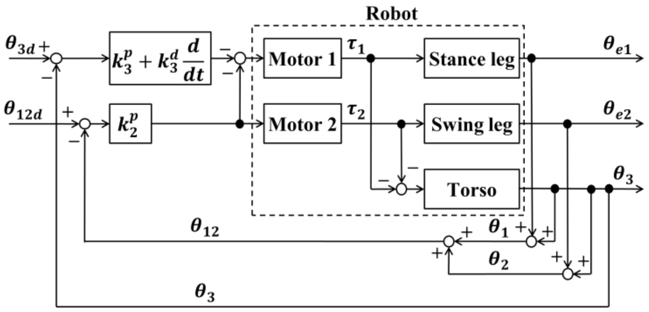

Fig. 11. Block diagram of an active control for experimental robot.

The total mass of the robot is 0.65[kg]. It has four legs in order to constrain the motion of the robot in the two dimensional plane. The inside legs and the outside legs are connected with each other, respectively. The control torques are loaded from the motors installed at the hip joints. The angles θ_{e1} and θ_{e2} of the legs are measured by using the encoders installed on the motors. The attitude angle θ_3 of the torso is measured by using the acceleration and angular velocity sensors attached on the torso.

VII. Conclusions

The biped walking robot with a torso had been studied. The walking motion of the robot on the horizontal ground was simulated by using the proportional or proportional-differential control law in the numerical calculations. The summaries of the results are:

(1) The reasonable control gains were proposed when the proportional or proportional-differential control was used.

(2) The larger control gains are needed when the desired attitude angle of the torso becomes larger.

(3) The attitude angle of the torso was stabilized by using the proportional-differential control rather than the proportional one.

(4) The difference between magnitudes of the control torques was not so large when the proportional or proportional-differential control was used. It means that the proportional-differential control was more effective than the proportional control in order to control the walking motion of the robot.

The experimental robot has been being developed considering the calculated results. After the development of the robot, the experiments on walking will be carried out as a future work.

Appendix

The components of matrixes and vector in Equation (1) are represented as

$$\boldsymbol{M}(\boldsymbol{\theta}) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & 0 \\ M_{13} & 0 & M_{33} \end{bmatrix}, \tag{a.1}$$

where

$$M_{11} = J_l + m_l a_l^2 + m_l l_l^2 + m_h l_l^2 + m_f l_l^2 + m_t l_l^2, \tag{a.2}$$
$$M_{12} = \{m_l l_l (l_l - a_l) + m_f l_l^2\} \sin(\theta_1 - \theta_2), \tag{a.3}$$
$$M_{13} = m_t a_t l_l \cos(\theta_1 - \theta_3), \tag{a.4}$$
$$M_{22} = J_l + m_l (l_l - a_l)^2 + m_f l_l^2 , \tag{a.5}$$
$$M_{33} = J_t + m_t a_t^2. \tag{a.6}$$
$$\boldsymbol{C}_C(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \begin{bmatrix} 0 & -C_{c12}\dot{\theta}_2 & C_{c13}\dot{\theta}_3 \\ C_{c12}\dot{\theta}_1 & 0 & 0 \\ -C_{c13}\dot{\theta}_1 & 0 & 0 \end{bmatrix}, \tag{a.7}$$

where

$$C_{c12} = \{m_l l_l (l_l - a_l) + m_f l_l^2\} \cos(\theta_1 - \theta_2), \tag{a.8}$$
$$C_{c13} = m_t a_t l_l \sin(\theta_1 - \theta_3). \tag{a.9}$$
$$\boldsymbol{C}_d = \begin{bmatrix} C_{d1} & 0 & 0 \\ 0 & C_{d2} & 0 \\ 0 & 0 & C_{d3} \end{bmatrix}, \tag{a.10}$$

where C_{d1}, C_{d2} and C_{d3} are constants.

$$\boldsymbol{g}(\boldsymbol{\theta}) = \{g_1 \quad g_2 \quad g_3\}^T, \tag{a.11}$$

where

$$g_1 = -g\left(m_l a_l + m_l l_l + m_h l_l + m_f l_l + m_t l_l\right) \times \sin\theta_1, \tag{a.12}$$
$$g_2 = g\left(m_l a_l - m_l l_l - m_f l_l\right)\cos\theta_2, \tag{a.13}$$
$$g_3 = -g m_t a_t \sin\theta_3. \tag{a.14}$$
$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}. \tag{a.15}$$

The components of matrixes in Equation (3) are represented as

$$\boldsymbol{Q}^+(\boldsymbol{\theta}^+) = \begin{bmatrix} Q_{11}^+ & Q_{12}^+ & Q_{13}^+ \\ Q_{21}^+ & M_{22} & 0 \\ Q_{31}^+ & 0 & M_{33} \end{bmatrix}, \tag{a.16}$$

where

$$Q_{11}^+ = M_{11} + Q_{21}^+ + Q_{31}^+, \tag{a.17}$$
$$Q_{12}^+ = M_{22} + Q_{21}^+, \tag{a.18}$$
$$Q_{13}^+ = M_{33} + Q_{31}^+, \tag{a.19}$$
$$Q_{21}^+ = \{m_l l_l (l_l - a_l) + m_f l_l^2\} \sin(\theta_1^+ - \theta_2^+), \tag{a.20}$$
$$Q_{31}^+ = m_t a_t l_l \cos(\theta_1^+ - \theta_3^+). \tag{a.21}$$
$$\boldsymbol{Q}^-(\boldsymbol{\theta}^-) = \begin{bmatrix} Q_{11}^- & Q_{12}^- & Q_{13}^- \\ Q_{21}^- & J_l & 0 \\ Q_{31}^- & 0 & M_{33} \end{bmatrix}, \tag{a.22}$$

where

$$Q_{11}^- = J_l - (2m_l a_l l_l + m_h l_l^2 + m_t l_l^2) \sin(\theta_1^- - \theta_2^-) + Q_{21}^- + Q_{31}^-, \tag{a.23}$$
$$Q_{12}^- = J_l + Q_{21}^-, \tag{a.24}$$
$$Q_{13}^- = M_{33} + m_t a_t l_l \sin(\theta_2^- - \theta_3^-), \tag{a.25}$$
$$Q_{21}^- = -m_l a_l (l_l - a_l), \tag{a.26}$$
$$Q_{31}^- = m_t a_t l_l \cos(\theta_1^- - \theta_3^-). \tag{a.27}$$

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