Numerical Quadratic Interpolation for Approximating Time of Death

Chinda Chaichuay[1,2], Wittaya Kongprasert[1], Apiwat Kruemuen[1], Chitchanok Mataynam[1] and Nopparat Pochai[1,2]

***Abstract*— Time is one of the most important factors of consideration in a murder case. It might delicately convict a murderer, break a alibi, alternately dispose of a suspect. If the circumstances surrounding death indicate the possibility of homicide, then both the body and immediate surrounding area become crucial in estimating time of death. Estimating the time of death, especially in cases where there are no witnesses, is critical to the investigation. The governing equation is governed by the Newton's law of cooling that provides the time of death. The Lagrange quadratic interpolation technique is use to fitting the experimental data so as to gives the accurately thermal conductivity. The traditional Newton method is also employed to approximate the solution. The proposed numerical technique gives good agreement estimating time of death.**

***Index Terms*— Newton's law of cooling, time of death, Lagrange interpolation, Newton method**

I. Introduction

TIME is one of the most important factors of consideration in a murder case. It may very well convict a murderer, break an alibi, or eliminate a suspect. If the circumstances surrounding death indicate the possibility of homicide, then both the body and immediate surrounding area become crucial in estimating time of death. Estimating the time of death, especially in cases where there are no witnesses, is critical to the investigation [4].

In [1], they propose an experiment in estimating the time of death (TOD). They were performed where a potato was used instead of a human body. The time of death for the potato was considered to be the time it was taken out of the oven. They show that the temperature of the potato at the time of death was 194 F. Temperature readings of the potato were taken every fifteen minutes for three hours. Estimates for the time of death at the corresponding temperatures were calculated with an average percent error of 7.29%. The ambient temperature was 75.2 F.

In this research, a mathematical model of the Newton's law of cooling that provides the time of death is introduced. The Lagrange quadratic interpolation technique is use to fitting the experimental data so as to gives the accurately thermal conductivity. The traditional Newton method is also employed to approximate the solution.

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[1]Department of Mathematics, Faculty of Science, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand (corresponding author e-mail: nop_math@yahoo.com).

[2]Centre of Excellence in Mathematics, Commission on Higher Education (CHE), Si Ayutthaya Road, Bangkok 10400, Thailand.

II. Time of Death Simulation

A. Time of death model

Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. To estimating the time of death, we require to obtain the temperature of the surroundings and the temperature of the victim body at two different times in order to make an precisely approximate. We assumed that the temperature of the victim body. The governing equation is governed by Newton's Law of Cooling [1, 3],

$$\frac{dT}{dt} = k(T(t) - T_s(t)) \qquad (1)$$

where k is a negative cooling constant (1/hr), $T_s(t)$ is the temperature of the surrounding (F), and time t is the number of hours since the time of death (hrs). Assume that the temperature of a human body at the time of death is regarded to be 98.60 F. It follows that $T(0) = 98.60$.

B. An investigation to approximate a cooling constant of a human body and time of death

Consider a crime scene in which a man is killed in his room. The body was found in the room early in the morning and at 7:00 A.M. The coroner measured its temperature, T_1 at that time. After one hour another temperature, T_2, was taken. The coroner noticed that the temperature of the murder victim's room was kept up at a consistent temperature of 70 F. The temperature of the surrounding in this case remained constant.

Let $T(t)$ be the temperature t hours after the body was $T_0 = 98.60$ F. The temperature of surrounding was a constant 70.00 F after the person's death. The coroner measured, $T_1 = 72.50$ at that time. After one hour another temperature, $T_2 = 72.00$. Newton's Law of cooling states becomes [1],

$$\frac{dT}{dt} = k(T - 70.00). \qquad (2)$$

We can obtain the solution of (1),

$$T(t) = T_s + (T_0 - T_s)e^{-kt}, \qquad (3)$$

which for the example is

$$T(t) = 70 + (98.60 - 70.00)e^{-kt}, \qquad (4)$$

Utilizing the two temperature estimations taken by the coroner, it follows that

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$$72.50 = 70.00 + 28.60e^{-kt_c}, \quad (5)$$

$$72.00 = 70.00 + 28.60e^{-k(t_c+1)}, \quad (6)$$

where t_c is the number of hours after death. The values for k and t_c can be obtained from Eqs.(5-6) as follow,

$$e^{-k} = \frac{T_1 - T_s}{T_2 - T_S} = 1.25. \quad (7)$$

It follows that $k = -0.2231\,\text{hr}^{-1}$. If we substituting k into Eq.(5) or Eq.(6), we can obtain that $t_c = 10.9219$ hrs. From this figured esteem it is trusted that the man is killed around 10.92 hours before 7:00 A.M., which would be around 8:00 P.M. the past night.

C. *Time of death experimentation*

In [1], they simulate another crime where a potato is the murder victim. Assume that the temperature of the potato is 194 F at the time of death so that *T(0)* = 194 F. Ensure the temperature of the room is kept consistent amid the test. Take various temperature readings of the potato after it dies. Approximate the time of death of the potato for every corresponding temperature and compare these approximates with the true values.

A potato will be utilized as a part of the place of a human body. The potato will be heated in an ordinary broiler and after that removed from the stove once a temperature of 194 F is acquired. Once the potato is removed from the broiler, it will be permitted to cool in a room where the temperature of the room is $T_s = 75.20$ F. Temperature readings of the potato will be taken at regular intervals keeping in mind the end goal to decide how well the prepared potato complies with Newton's Law of cooling,

$$\begin{aligned} T(t) &= 75.20 + (194 - 75.20)e^{-kt}, \\ &= 75.20 + 118.8e^{-kt}. \end{aligned} \quad (8)$$

III. Quadratic Interpolation to Approximate Time of Death

A. *The quadratic interpolation*

Assume that there is a collection of $M+1$ data points,

$$(x_0, y_0), \dots, (x_j, y_j), \dots, (x_M, y_M), \quad (9)$$

and there are no two x_j are equal. The interpolation polynomial in the Lagrange form is a linear combination as [2],

$$L(x) = \sum_{j=0}^{M} y_j l_j(x), \quad (10)$$

where Lagrange basis polynomials,

$$l_j(x) = \prod_{\substack{0 \le m \le M \\ m \ne M}} \frac{n - x_m}{x_j - x_m}, \quad (11)$$

where $0 \le j \le M$. If $M = 2$, the interpolation is called a quadratic interpolation.

Temperature readings of the potato will be taken at regular intervals with a specific end goal to decide how well the heated potato complies with Newton's Law of cooling with quadratic interpolation. The slope of the line in the semi-log plot corresponds to the constant k. We can get an interpolated function per three data points, so we also obtain a calculated cooling constant k_i for each subinterval. After we obtain all interpolated functions, so we also obtain k_i as well. Consequently, the averaged cooling constant can be obtain by $k_I = \sum_{i=1}^{8} k_i$, where $I = 1, 2, \dots, 6$.

B. *Numerical results*

We can see the calculated cooling constants in Tables I-II. The measured cooling temperatures of the potato over calculated times are shown in Tables III-IV and Fig. 1.

TABLE I
THE AVERAGED COOLING CONSTANT BY USING INTERPOLATION ON THE POTATO TEMPERATURE OVER A TIME INTERVAL

Time (min)	Interpolated values	Averaged cooling constant (min^{-1})
t	K(t)	K_I
0	-0.00333	
5	-0.00555	
10	-0.00777	
15	-0.00999	-0.02167
20	-0.01221	
25	-0.01443	
30	-0.01665	

TABLE II
THE AVERAGED COOLING CONSTANT BY USING INTERPOLATION ON THE POTATO TEMPERATURE OVER A TIME INTERVAL

Time interval (min)	Interpolated cooling constant (min^{-1})
t	K_I
0-30	-0.02167
30-60	-0.01338
60-90	-0.01344
90-120	-0.01012
120-150	-0.00667
150-180	-0.01334

TABLE III
THE COOLING OF THE POTATO OVER TIME

Temperature (F)	Time of death (min)		
	Measured	Regression method [1]	Quadratic interpolation
194.00	0.00	0.00	0.00
183.60	7.90	15.00	7.52
162.50	26.56	30.00	25.28
144.30	46.71	45.00	44.47
132.80	62.41	60.00	59.41
122.20	79.94	75.00	76.09
115.90	92.35	90.00	87.90
109.20	107.85	105.00	102.66
103.60	123.37	120.00	117.43
99.00	138.60	135.00	131.93
97.70	143.44	150.00	136.54
94.30	157.56	165.00	149.99
90.50	176.69	180.00	168.19

IV. Conclusion

The investigation was fruitful. The information that was taken and fitted to condition (3) complied with Newton's Law of Cooling genuinely well. The quadratic interpolation is used to estimate an accurately cooling constant. The proposed technique gives a good agreement approximated time of death due to the cooling constant is obtained by using many data points. Therefore, the frequently victim body temperature measurement is required.

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TABLE IV
THE ABSOLUTE ERROR OF APPROXIMATED COOLING CONSTANT WITH THE TRUE VALUE BY USING REGRESSION AND QUADRATIC INTERPOLATION FOR EACH TIME INTERVALS

Time of death (min)	Absolute error	
	Regression[1]	Interpolation
0	0.0000	0.0000
15	8.5727	2.9342
30	3.4155	5.3771
45	1.6308	0.1329
60	1.6308	0.1329
75	2.7714	3.5056
90	1.1228	2.8238
105	1.1436	1.0155
120	1.1310	1.1863
135	1.0148	1.8535
150	1.6482	1.0722
165	1.5783	0.8066
180	0.5766	0.4267

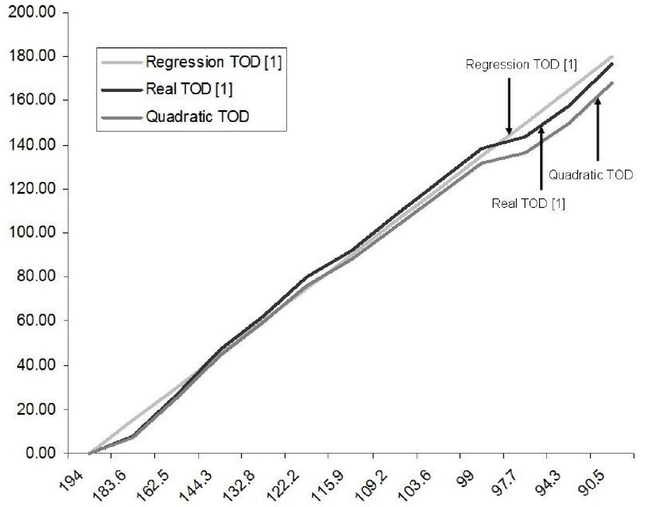

Fig. 1. Comparison of time of death by measurement, regression method and quadratic interpolation method