Mixed Moving Average-Exponentially Weighted Moving Average Control Charts for Monitoring of Parameter Change

Rattikarn Taboran, Saowanit Sukparungsee, and Yupaporn Areepong

Abstract—**Statistical Process Control (SPC) is the popular tool for controlling the statistical process to improve the quality production processes, and to monitor the alteration as soon as possible. Typically, the Shewhart control chart detects the large change in the mean of production processes whereas the MA and EWMA control charts detect a small change. This research aimed to propose the new control chart: Moving Average-Exponentially Weighted Moving Average's control chart (MA-EWMA) to detect a change in process mean underlying asymetrics and symmetries processes, and compare the efficiency in monitoring the change with Shewhart, EWMA, and MA control chart at the parameter change levels. Efficiency criteria was the Average Run Length (ARL) which evaluated by using Monte Carlo Simulation (MC) for MA-EWMA and EWMA charts and by using the explicit formula of ARL for Shewhart and MA charts. The numerical results showed that MA-EWMA had better performance than Shewhart, EWMA, and MA control charts for all parameter change levels.**

Index Terms—**Mixed control chart, Average Run Length, Moving Average-Exponentially Weighted Moving Average control chart, Moving Average control chart**

I. Introduction

At present, the quality control of production processes in industrial factories or operational facilities is greatly important due to the high levels of competition in the markets. Therefore, these operational facilities need to apply the statistical quality control tools in order to detect and regulate any changes in these processes. The tool having the most efficiency in detecting the changes occurring in processes is the control chart, as the results can be shown clearly, and thus it is popular and widely used.

In 1931, W.A. Shewhart [1] developed the control chart for the first time and divided into control charts for variables and attributes. The control charts mentioned above use the principles of constructing the control limits from Shewhart, called the 3σ control charts, or the standard control charts. These charts are able to effectively detect changes in the mean of production processes and the process of major changes $(\delta > 1.5\sigma)$. As the standard control charts do not include past data, therefore, control charts that account for past data, such as the cumulative sum control chart (CUSUM) proposed by Page [2], were invented. Later, Roberts [3] proposed the EWMA chart, which is effective in detecting the minor variations of processes well $(\delta \leq 1.5\sigma)$ by Montgomery [4]. After that, in 1994, Butler and Stefani [5] proposed the Double Exponentially Weighted Moving Average control chart (DEWMA). Later, Khoo [6] developed the Moving Average control chart (MA), which is a control chart calculating the average by finding the moving average (w). It can detect minor changes well and can be used for both continuing and discontinuing distribution. Later, in 2008, Khoo and Wong [7] jointly developed the Double Moving Average control chart (DMA), which is a chart controlling the statistical value of the MA chart to find the moving value again. In order to show the efficiency of the DMA chart compared to the MA chart, CUSUM chart, and EWMA chart by using the Monte Carlo method, it was found that when a small change in the process ($\delta \leq 0.1$) occurs, the EWMA and CUSUM charts will efficiently detect the best changes. When a moderate changing process ($\delta \leq 0.1$) occurs, ($0.2 \leq \delta \leq 1.5$), the DMA control chart will have the best efficiency. In 2009, Sukparungsee and Areepong [8] studied the performance of EWMA chart with transformed Weibull observations, and compared the performance of EWMA versus CUSUM charts are considered, the performance of EWMA chart is superior to CUSUM for small changes. On the contrary, the performance of EWMA chart is inferior to CUSUM chart for moderate to large changes. After that, in 2012, Abbas, Riaz, and Does [9] proposed a mixed EWMA-CUSUM control chart for detecting a shift in the process mean and evaluated its average run lengths. The comparisons revealed that mixing the two charts makes the proposed scheme even more sensitive to the small shifts in the process mean than the other schemes designed for detecting small shifts. Phantu, Sukparungsee, and Areepong [10] studied explicit expression of ARL of MA control chart for Poisson integer valued autoregressive model, The results showed that MA chart performs better than others when the

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magnitudes of shift are moderate and large. Later, The mixed CUSUM-EWMA chart which is used to monitor the location of a process better than as proposed by [11]. Recently, a new control chart: Double Moving Average-EWMA control chart for exponentially distributed quality was presented by [12]. The efficiency of this control chart showed that it is better than existing chart when the shift size is very small.

This research proposed the new control chart, namely, MA-EWMA and compared the efficiency in monitoring the parameter change to Shewhart control chart, EWMA, and MA control chart considering from ARL. The control chart that had quick detection the change in process mean was the most efficient control chart.

II. Moving Average (MA), Exponentially Weighted Moving Average (EWMA), Mixed MA-EWMA, and the Proposed Mixed MA-EWMA Control Charts

In this section, we consider control charts that also use previous observations along with the current observation. These mainly include MA and EWMA schemes, and we provide here the details regarding their usual design structures (also known as classical MA and EWMA control charts).

A. Moving Average (MA) control chart

MA control chart is the suitable chart for detecting the small change. The MA was measured at the each period (w). Assume that individual measurements $X_1, X_2,...$ where $X_i \sim N(\mu,\sigma^2)$, for $i = 1,2,...$ are obtained from a process. The MA statistic of span *w* at time *i* is defined by Montgomery [4] as follow

$$MA_i = \begin{cases} \dfrac{X_i + X_{i-1} + X_{i-2} + ...}{i} & ,i < w \\ \dfrac{X_i + X_{i-1} + ... + X_{i-w+1}}{w} & ,i \ge w. \end{cases} \quad (1)$$

The average of all measurements up to period i defines the MA. The mean and variance of the moving average statistic, MA_i are:

$$E(MA_i) = E(X_i) = \mu_0 \quad (2)$$

and

$$Var(MA_i) = \begin{cases} \dfrac{\sigma^2}{i} & ,i < w \\ \dfrac{\sigma^2}{w} & ,i \ge w. \end{cases} \quad (3)$$

where μ_0 denotes the in-control value of the process mean. The control limits of the MA chart are:

$$UCL/LCL = \begin{cases} \mu_0 \pm \dfrac{K_1\sigma}{\sqrt{i}} & ,i < w \\ \mu_0 \pm \dfrac{K_1\sigma}{\sqrt{w}} & ,i \ge w. \end{cases} \quad (4)$$

B. Exponentially Weighted Moving Average (EWMA) control chart

EWMA control chart proposing by Roberts [3] is the chart to quickly detect the parameter change when the small change occurs in the process. The statistic of EWMA chart are:

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1} \quad ,i = 1,2,\ldots \quad (5)$$

where X_i denoted the value from the processes with normal distribution, which the mean is μ_0, the variance is σ^2, and λ is the weighted moving average, where $0 \le \lambda \le 1$. The mean and variance of the EWMA statistic are as follows:

$$E(Z_i) = \mu_0 \quad (6)$$

and

$$Var(Z_i) = \sigma^2 \left\{ \frac{\lambda}{2-\lambda} \left(1-(1-\lambda)^{2i}\right) \right\} \quad (7)$$

The time-varying control limits of the EWMA statistic are given as:

$$\begin{aligned} LCL &= \mu_0 - K_2 \sqrt{\sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \\ UCL &= \mu_0 + K_2 \sqrt{\sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}. \end{aligned} \quad (8)$$

where $i = 1,2,3,4,\ldots n$ and K_2 is the control limit coefficient of EWMA that is specified according to a pre-specified false alarm rate or average run length (ARL) when the process is assumed in control (i.e., ARL_0). Moreover, as $i \to \infty$, $(1\text{-}\lambda)^{2i} \to 0$, the variance of the EWMA statistic becomes $Var(Z_i) = \sigma^2(\lambda/(2-\lambda))$, in the steady state, control limits of the EWMA chart can be defined as:

$$\begin{aligned} LCL &= \mu_0 - K_2 \sqrt{\sigma^2 \left(\frac{\lambda}{2-\lambda}\right)} \\ UCL &= \mu_0 + K_2 \sqrt{\sigma^2 \left(\frac{\lambda}{2-\lambda}\right)}. \end{aligned} \quad (9)$$

C. Mixed MA-EWMA control chart

At the combination of MA and EWMA control chart, the statistic is that of the MA control chart, as shown in Equation 1. The upper control limit (UCL) and lower control limit (LCL) value of MA-EWMA control chart is the data expectation value, which is the same value with that of EWMA control chart. The variance is applied with the combination of MA and EWMA control chart, as shown in Equation 3 and 7.

The average of all measurements up to period *i* defines the MA. Now based on the EWMA design, the time-varying control limits for MA-EWMA can be defined in the form, namely LCL and UCL given as:

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$$LCL = \mu_0 - K_3\sqrt{\left(\frac{\sigma^2}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]} \quad \text{for } i \geq w \qquad (10)$$
$$UCL = \mu_0 + K_3\sqrt{\left(\frac{\sigma^2}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]}.$$

where μ_0 is the target value of the mean. The limits for periods $i < w$ are obtained by replacing $\frac{\sigma^2}{w}$ with $\frac{\sigma^2}{i}$ in Equation (10). Where $i = 1,2,3,4,\ldots n$ and K_3 is the control limit coefficient of MA-EWMA that is specified according to a pre-specified false alarm rate or average run length (ARL) when the process is assumed in control (ARL_0). Moreover, as $i \to \infty$, $(1-\lambda)^{2i} \to 0$, in the steady state, control limits of the MA-EWMA chart can be defined as:

$$LCL = \mu_0 - K_3\sqrt{\left(\frac{\sigma^2}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)} \quad \text{for } i \geq w \qquad (11)$$
$$UCL = \mu_0 + K_3\sqrt{\left(\frac{\sigma^2}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)}.$$

the limits for periods $i < w$ are obtained by replacing $\frac{\sigma^2}{w}$ with $\frac{\sigma^2}{i}$ in Equation (11).

III. Performance Measures Evaluation

Average Run Length (ARL) is the indicator of the control chart efficiency to detect the quantity in the production processes. This is considered from the quickness of detecting the value outside the control when the mean of the process changes. Any control chart that quicker detects the change in the production process is the efficient chart. This research studied the two methods of finding ARL: the explicit formula by Areepong [13] for Shewhart and MA control chart, and finding ARL with MC, which is the created program to find ARL as follows.

$$ARL = \frac{\sum_{t=1}^{N} RL_t}{N} \qquad (12)$$

where RL_t is the monitored observation data before discovering that the process is out of the control for the first time of the data simulation *t* and N is the number of the experiment repetition. Finding ARL of the EWMA and MA-EWMA control chart can be done by applying MC method, where K, K_1, K_2, K_3 are the coefficient of the limits of each control chart, and the values are set as follows: 1) Set the sample size (m) of each round of experiment at 10,000, 2) Set the number of the experiment repetition (N) at 200,000, and 3) Set $\text{ARL}_0 = 370$ when the process is under control.

IV. Performance Analysis and Comparisons

This research studied the efficiency in monitoring the change resulted from ARL when the processes were not under control. The study was under the five distributions processes, which were symmetrical distribution: Normal(0,1), Laplace(0,1), and Logistic(6,2), and non-symmetrical distribution, skew to the right: Exponential(1) and Gamma(4,1), that compared the efficiency in detecting the change of the control chart when the change was $\delta \in [-4,4]$ as shown in the following table.

From Table I and Fig 1, the normal data distribution which the parameter was $\mu = 0, \sigma^2 = 1$ and $K_3 = 7.632$ (where $w = 5, \lambda = 0.25$) of the MA-EWMA control chart showed that ARL_1 was lower than that of the MA, EWMA, and Shewhart chart at all change levels. Only the change at -0.50, 0.50, 0.75 level that the MA chart had lower than MA-EWMA chart, however, the value was slightly different.

From Table II, Laplace distribution set the parameter $\alpha = 0, \beta = 1$, MA-EWMA showed that $K_3 = 8.242$ (where $w = 5, \lambda = 0.25$) and ARL_1 was lower than MA, EWMA, and Shewhart control charts at all change levels. MA-EWMA control charts had better efficiency in monitoring parameter change than other control chart. If parameter value increased or decreased, ARL_1 resulted showed in Fig 1

From Table III, Logistic distribution set the parameter $\mu = 6, s = 2$, MA-EWMA showed that $K_3 = 7.632$ (where $w = 5, \lambda = 0.25$) and ARL_1 was lower than MA, EWMA, and Shewhart control charts at all change levels. MA-EWMA control charts had better efficiency in monitoring parameter change than other control chart. If parameter

TABLE I
ARL performance of Shewhart, MA, EWMA, and MA-EWMA for Normal(0,1).

δ	*w=5, λ=0.25*			
	$K=3.000$ *Shewhart*	$K_1=3.000$ *MA*	$K_2=2.927$ *EWMA*	$K_3=7.632$ *MA-EWMA*
-4.00	1.19	1.16	1.08 ± 0.01	*0.00 ± 0.00*
-3.00	2.00	1.62	1.38 ± 0.01	*0.05 ± 0.00*
-2.00	6.30	2.81	2.51 ± 0.03	*0.47 ± 0.00*
-1.50	14.97	3.84	4.19 ± 0.06	*1.51 ± 0.01*
-1.00	43.89	6.97	9.09 ± 0.15	*5.83 ± 0.02*
-0.75	81.22	*13.15*	16.72 ± 0.31	13.52 ± 0.04
-0.50	155.22	*35.51*	39.93 ± 0.35	37.18 ± 0.11
-0.25	281.15	134.51	136.26 ± 0.48	*124.59 ± 0.35*
-0.10	352.93	296.16	307.28 ± 0.62	*243.70 ± 0.68*
-0.05	365.89	348.91	351.09 ± 0.67	*277.22 ± 0.77*
0.00	370.40	370.40	370.00 ± 0.69	370.05 ± 0.83
0.05	365.89	348.91	344.66 ± 0.65	*276.83 ± 0.77*
0.10	352.93	296.16	302.15 ± 0.60	*242.74 ± 0.68*
0.25	281.15	134.51	138.84 ± 0.47	*124.39 ± 0.35*
0.50	155.22	*35.51*	40.76 ± 0.35	37.20 ± 0.11
0.75	81.22	*13.15*	16.70 ± 0.33	13.52 ± 0.04
1.00	43.89	6.97	9.11 ± 0.16	*5.87 ± 0.02*
1.50	14.97	3.84	4.18 ± 0.06	*1.53 ± 0.01*
2.00	6.30	2.81	2.54 ± 0.03	*0.47 ± 0.00*
3.00	2.00	1.62	1.39 ± 0.01	*0.05 ± 0.00*
4.00	1.19	1.16	1.08 ± 0.01	*0.00 ± 0.00*

Note that: the italic was minimum of ARL and after the mark $(\pm)$ was standard deviation of ARL

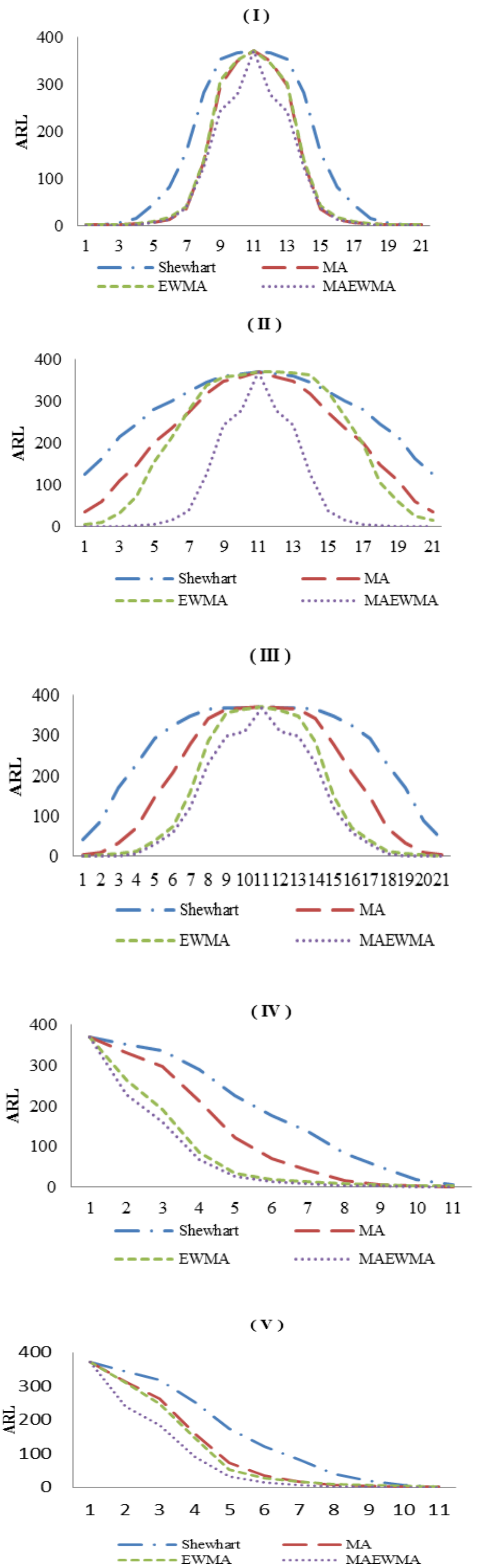

Fig 1. ARL curves of Shewhart, MA, EWMA and MA-EWMA for I) Normal(0,1) distribution; II) Logistic(6,2) distribution; III) Laplace (0,1) distribution; IV) Exponential(1) distribution; and V) Gamma(4,1) distribution.

TABLE II
ARL PERFORMANCE OF SHEWHART, MA, EWMA, AND MA-EWMA FOR LAPLACE (0,1) .

δ	w=5, λ =0.25			
	$K=5.915$ *Shewhart*	$K_1=5.915$ *MA*	$K_2=3.396$ *EWMA*	$K_3=8.242$ *MA-EWMA*
-4.00	43.65	4.50	1.79 $\pm$ 0.02	*0.08 $\pm$ 0.00*
-3.00	87.58	9.26	2.93 $\pm$ 0.03	*0.34 $\pm$ 0.00*
-2.00	170.12	33.60	6.88 $\pm$ 0.10	*2.37 $\pm$ 0.01*
-1.50	229.13	70.45	14.01 $\pm$ 0.25	*7.94 $\pm$ 0.02*
-1.00	293.95	147.59	38.94 $\pm$ 0.29	*29.32 $\pm$ 0.08*
-0.75	323.93	208.00	74.36 $\pm$ 0.33	*59.36 $\pm$ 0.16*
-0.50	348.54	279.30	159.05 $\pm$ 0.35	*122.07 $\pm$ 0.33*
-0.25	364.84	343.52	287.64 $\pm$ 0.58	*232.58 $\pm$ 0.61*
-0.10	369.630	365.995	357.64 $\pm$ 0.67	*298.00 $\pm$ 0.78*
-0.05	370.323	369.406	365.47 $\pm$ 0.69	*310.33 $\pm$ 0.82*
0.00	370.554	370.554	370.42 $\pm$ 0.86	370.17 $\pm$ 0.84
0.05	370.323	369.406	362.05 $\pm$ 0.68	*310.87 $\pm$ 0.82*
0.10	369.630	365.995	349.88 $\pm$ 0.66	*299.15 $\pm$ 0.79*
0.25	364.84	343.52	283.41 $\pm$ 0.58	*233.04 $\pm$ 0.61*
0.50	348.54	279.30	150.03 $\pm$ 0.40	*121.98 $\pm$ 0.33*
0.75	323.93	208.00	72.00 $\pm$ 0.32	*59.19 $\pm$ 0.16*
1.00	293.95	147.59	38.23 $\pm$ 0.28	*29.23 $\pm$ 0.08*
1.50	229.13	70.45	13.95 $\pm$ 0.24	*7.92 $\pm$ 0.02*
2.00	170.12	33.60	6.69 $\pm$ 0.10	*2.37 $\pm$ 0.01*
3.00	87.58	9.26	2.88 $\pm$ 0.03	*0.34 $\pm$ 0.00*
4.00	43.65	4.50	1.77 $\pm$ 0.02	*0.08 $\pm$ 0.00*

Note that: the italic was minimum of ARL and after the mark ($\pm$) was standard deviation of ARL

TABLE III
ARL PERFORMANCE OF SHEWHART, MA, EWMA, AND MA-EWMA FOR LOGISTIC (6,2).

δ	w=5, λ =0.25			
	$K=17.828$ *Shewhart*	$K_1=17.828$ *MA*	$K_2=3.298$ *EWMA*	$K_3=7.632$ *MA-EWMA*
-4.00	123.86	34.42	3.56 $\pm$ 0.03	*0.00 $\pm$ 0.02*
-3.00	162.83	60.89	8.79 $\pm$ 0.09	*0.05 $\pm$ 0.04*
-2.00	214.12	110.09	31.52 $\pm$ 0.11	*0.47 $\pm$ 0.11*
-1.50	245.55	148.75	71.99 $\pm$ 0.25	*1.51 $\pm$ 0.20*
-1.00	281.59	201.39	153.76 $\pm$ 0.46	*5.83 $\pm$ 0.37*
-0.75	301.55	234.45	213.53 $\pm$ 0.60	*13.52 $\pm$ 0.50*
-0.50	322.91	272.99	278.33 $\pm$ 0.71	*37.18 $\pm$ 0.65*
-0.25	345.78	317.92	338.66 $\pm$ 0.73	*124.59 $\pm$ 0.77*
-0.10	360.271	348.37	357.46 $\pm$ 0.76	*243.70 $\pm$ 0.78*
-0.05	365.233	359.15	363.75 $\pm$ 0.78	*277.22 $\pm$ 0.79*
0.00	370.264	370.26	370.12 $\pm$ 0.81	370.05 $\pm$ 0.83
0.05	365.233	359.15	370.99 $\pm$ 0.80	*276.83 $\pm$ 0.79*
0.10	360.271	348.37	368.94 $\pm$ 0.79	*242.74 $\pm$ 0.78*
0.25	345.78	317.92	364.44 $\pm$ 0.78	*124.39 $\pm$ 0.77*
0.50	322.91	272.99	324.30 $\pm$ 0.72	*37.20 $\pm$ 0.65*
0.75	301.55	234.45	262.91 $\pm$ 0.62	*13.52 $\pm$ 0.50*
1.00	281.59	201.39	197.35 $\pm$ 0.48	*5.87 $\pm$ 0.37*
1.50	245.55	148.75	103.97 $\pm$ 0.24	*1.53 $\pm$ 0.20*
2.00	214.12	110.09	60.00 $\pm$ 0.13	*0.47 $\pm$ 0.11*
3.00	162.83	60.89	24.91 $\pm$ 0.11	*0.05 $\pm$ 0.04*
4.00	123.86	34.42	13.58 $\pm$ 0.09	*0.00 $\pm$ 0.02*

Note that: the italic was minimum of ARL and after the mark ($\pm$) was standard deviation of ARL

value increased or decreased, ARL_1 resulted showed in Fig 1

From Table IV, the Exponential distribution set the parameter $\lambda = 1$, MA-EWMA showed that $K_3 = 4.424$

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(where $w=5, \lambda=0.25$), and ARL_1 was lower than MA, EWMA, and Shewhart control charts at all change levels. MA-EWMA control charts had better efficiency in monitoring parameter change than other control chart, ARL_1 resulted showed in Fig 1

From Table V, Gamma distribution set the parameter $\alpha=4, \beta=1$, MA-EWMA showed that $K_3=2.0005$ (where $w=5, \lambda=0.25$) and ARL_1 was lower than MA, EWMA, and Shewhart control charts at all change levels. MA-EWMA control charts had better efficiency in monitoring parameter change than other control chart, ARL_1 resulted showed in Fig 1

TABLE IV
ARL PERFORMANCE OF SHEWHART, MA, EWMA, AND MA-EWMA FOR EXPONENTIAL(1).

δ	*w=5, λ=0.25*			
	$K=3.000$ *Shewhart*	$K_1=3.000$ *MA*	$K_2=3.747$ *EWMA*	$K_3=4.424$ *MA-EWMA*
0.00	370.55	370.18	370.38 ± 0.86	370.28 ± 0.84
0.05	352.482	331.16	263.70 ± 0.74	*228.20 ± 0.57*
0.10	335.291	296.28	192.00 ± 0.66	*160.43 ± 0.40*
0.25	288.59	212.29	85.58 ± 0.52	*67.96 ± 0.18*
0.50	224.75	122.15	33.73 ± 0.46	*25.41 ± 0.07*
0.75	175.04	70.74	19.55 ± 0.36	*13.04 ± 0.04*
1.00	136.32	41.44	13.54 ± 0.24	*8.03 ± 0.02*
1.50	82.68	15.32	7.99 ± 0.14	*4.11 ± 0.01*
2.00	50.15	6.96	5.62 ± 0.09	*2.59 ± 0.01*
3.00	18.45	3.27	3.64 ± 0.06	*1.40 ± 0.01*
4.00	6.79	2.08	2.76 ± 0.04	*0.93 ± 0.00*

Note that: the italic was minimum of ARL and after the mark $(\pm)$ was standard deviation of ARL

TABLE V
ARL PERFORMANCE OF SHEWHART, MA, EWMA, AND MA-EWMA FOR GAMMA(4,1).

δ	*w=5, λ=0.25*			
	$K=3.000$ *Shewhart*	$K_1=11.786$ *MA*	$K_2=3.549$ *EWMA*	$K_3=2.0005$ *MA-EWMA*
0.00	370.31	370.03	370.04 ± 0.87	370.29 ± 0.84
0.05	342.96	311.96	311.13 ± 0.80	*239.33 ± 0.63*
0.10	317.67	263.29	247.13 ± 0.67	*184.63 ± 0.49*
0.25	252.75	159.46	145.89 ± 0.45	*89.16 ± 0.24*
0.50	173.33	71.27	53.20 ± 0.34	*31.97 ± 0.09*
0.75	119.48	33.63	25.79 ± 0.33	*13.92 ± 0.04*
1.00	82.82	17.22	16.17 ± 0.27	*6.99 ± 0.02*
1.50	40.56	6.52	8.50 ± 0.10	*2.31 ± 0.01*
2.00	20.45	4.07	5.87 ± 0.06	*0.93 ± 0.00*
3.00	5.84	2.30	3.49 ± 0.03	*0.20 ± 0.00*
4.00	2.10	1.53	2.46 ± 0.02	*0.04 ± 0.00*

Note that: the italic was minimum of ARL and after the mark $(\pm)$ was standard deviation of ARL

V. CONCLUSION

This research purposed the new control chart that was the combination of MA and EWMA control chart called MA-EWMA control chart that studied the MA-EWMA control chart underlying symetrics distribution: Normal(0,1), Laplace(0,1), and Logistic(6,2), and asymmetries distribution, skew to the right Exponential(1), and Gamma(4,1). Findings illustrated that MA-EWMA had better performance than MA, EWMA, and Shewhart charts at all change levels in distributed data Laplace(0,1), Logistic(6,2), Exponential(1), and Gamma(4,1) but distributed data Normal(0,1) showed that MA-EWMA had better performance than MA, EWMA, and Shewhart chart at all change levels. Only the change at -0.50, 0.50, 0.75 level that the MA chart had lower ARL_1 than MA-EWMA chart, however, the value was slightly different. For the research in the future, other distributions may be applied and the other methods to find ARL or other control setting may be added.

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