# Energy Efficiency Analysis of a Scroll-type Air Motor Based on a Simplified Mathematical Model

Li Yang, Jihong Wang, Nan Lu, Stephen Mangan, James W Derby

***Abstract*—The paper presents a simplified mathematical model for a scroll-type air motor. Through air power/energy analysis, it has shown that a scroll-type air motor with a properly specified structure and suitable pressure of compressed air supply is able to achieve high energy efficiency and satisfies air power requirement. The features of a scroll-type air motor in geometry, mechanical structure, and pneumatic power transmission are studied in the paper. Differential geometry is used as the mathematical tool to parameterise the scroll. In this study, Matlab Symbolic Math Toolbox is found very efficient in finding analytic expressions of scroll chamber volumes and generalised torques. State shifting is employed to solve the dynamic system equations numerically. Energy efficiency of a scroll-type air motor is analysed based on the mathematical model.**

***Index Terms*— Energy efficiency, scroll-type air motors, pneumatic actuators, mathematical modeling.**

## I. Introduction

Since 1960s, compressed air, as a kind of clean and safe energy, has been used in modern industrial manufacturers. Compared with hydraulic and electrical counterparts, pneumatic systems are environment friendly, cleaner, simpler, easier in maintenance and safer. They can work in harsh environments, work without sparks, stall without damages. Nowadays, compressed air systems take up a significant part of the total electricity consumption in manufacturing industry. In 1998, in Japan, pneumatic systems consumed 10% to 20% of the total electricity supplied to factories, reaching 40 billion kilowatt hour [1]. That was approximately 5% of the national total electricity consumed in Japan [2].

Customers are becoming aware of that pneumatic actuators are comparatively expensive to operate due to its lower energy efficiency. In fact, energy efficiency of pneumatic actuators is found to be 23~30% and often lower than 20%, which is much lower than its electrical counterparts, 60% [2]. Users have realised that some irreversible processes, such as air leaks, frictions, pressure losses through dryers, filters, etc as well as improper settings and operation account for energy waste of pneumatic systems.

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A scroll type air motor, also known as a scroll expander is a relatively new to pneumatic actuators. Its unique structure features it many advantages as well as higher ability of energy conversion than conventional pneumatic actuators, such as cylinders, vane-type air motors, and *etc*. The scroll technique is now mainly and widely implemented in air conditioner and refrigerator compressors, because its compact design matches the requirement of small, quiet, and highly efficient refrigeration compressors. Recently, the concept was re-invented to air motors. In stead of compressing air to raise pressure, the air motors or expanders release the high pressure air energy to generate a driving force, which leads to a suitable actuator for different applications. For example, scroll air motors can be used to recover work in fuel cells [4].

This paper aims to explain why scroll air motors have higher energy efficiency than other type of air motors. The fundamental progress to the explanation is a newly developed simplified mathematical model of a scroll-type air motor. The model starts from the basic geometry description to the description of dynamic relationships of compressed air, generalised torques, energy conversion, and dynamic responses. The paper reports a simplified mathematical model and its energy efficiency analysis based on the proposal of air power calculation given in [1] and [2].

## II. Working Processes of a Scroll Air Motor

A scroll-type air motor is essentially a refrigeration scroll compressor working backwards. It consists of two intermeshed identical scrolls, one of which is rotated through 180 degrees with respect to the other, that is, one is mirrored with respect to the other [5]. The moving scroll can revolve eccentrically with respect to the fixed one to form several sealed crescent chambers. The moving scroll wobbles inside the fixed scroll, which does not rotate but just wobbles on a cam, where a shaft is located. Compressed air is introduced into the centre of a scroll air motor through the inlet, then the potential energy forces the moving scroll wobble in the direction that the chambers are getting bigger and towards the outer periphery to drive the crankshaft. This structure features scroll air motors desirable advantages relative to other types of air motors, because the motion is rotary and can be completely balanced

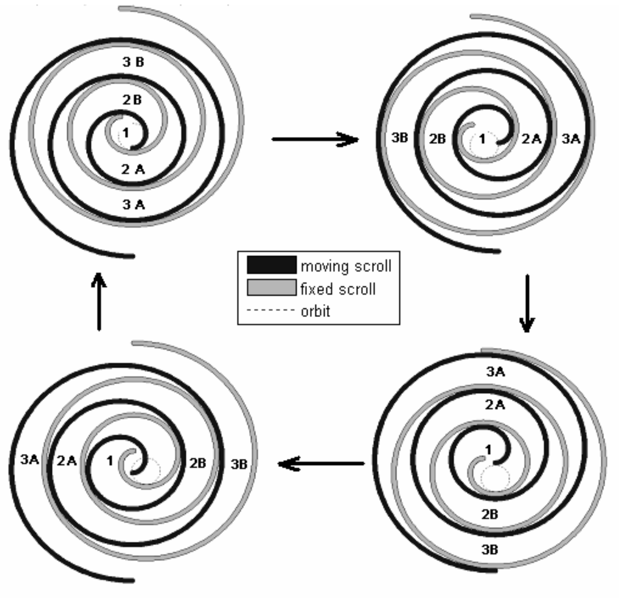


Fig. 1 Schematic diagram of a scroll-type air motor

which will reduce vibrations and noise. All these characteristics make a scroll air motor a simple, quiet and reliable machine at a competitive manufacturing cost.

A typical scroll air motor in motion is shown in Fig. 1. In the figure, both scrolls are circular involutes of three wraps. The grey scroll is the fixed scroll and the black one represents the moving scroll that travels along the orbit anticlockwise when compressed air comes into Chamber 1 in the centre through the inlet. Each scroll is fitted to a back plate. As the moving scroll travels along the circular orbit, these two scrolls keep contacting at some points, so that there are even number of crescent chambers.

When a scroll air motor is running, it goes successively and periodically through the four states in the sequence indicated by the arrows as shown in Fig. 1. An air motor working process includes three phases, charging, expansion, and discharging phase. A charging phase starts at the upper left diagram in Fig. 1, when compressed air starts entering the central chamber marked with “1”. There are now four sealed chambers, “2A”, “2B”, “3A”, and “3B”. After one quarter of a cycle, the scroll air motor comes to the upper right diagram in Fig. 1, the chamber “1” is getting bigger; chambers “2A”, “2B”, “3A”, and “3B” have moved anticlockwise and increased in size. It continues through another to quarters of cycle. Then after one complete cycle, the air motor comes back to the diagram in upper right. But the gas started life in chamber “1” is now partially chambers “2A” and “2B” and get sealed. After the air is sealed, it starts to expand. As the air motor continues through the second cycle, the air in chambers “2A” and “2B” expands and ends up going into chambers “3A” and “3B”. Through the third cycle, the air in chambers “3A” and “3B” expands further and finally discharges to atmosphere when “3A” and “3B” are not sealed any more, immediately after the state shown in the lower right diagram in Fig. 1. Compressed air always pushes the moving scroll to go along the orbit through the three phases so that there is output work delivered through the eccentric shaft.

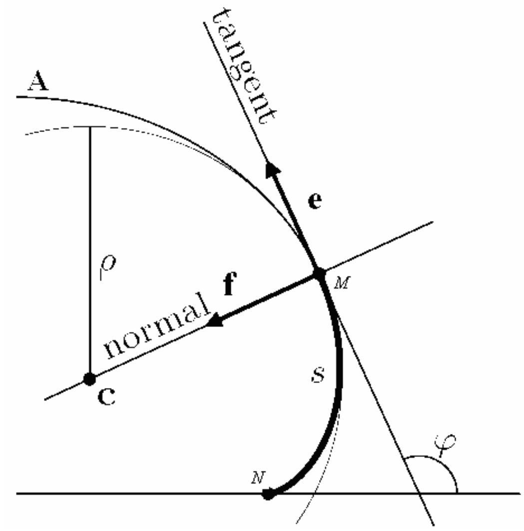


Fig. 2 Basic geometry of a spiral [6]

## III. Geometry of a Scroll Air Motor

### *A. The mathematical description for a spiral*

A spiral is the fundamental geometry curve of a scroll. A circular spiral is shown in Fig. 2, in which $\varphi$ is the tangential angle of a point on Spiral $\mathbf{A}$. Tangential angle is the angle formed from the horizon to the tangent line at a point on the curve. $s$ represents the length of the arc $MN$, $\mathbf{e}(\varphi)$ is the unit tangent vector at point $M$, $\mathbf{f}(\varphi)$ is the unit normal vector at point $M$, $\mathbf{f}(\varphi)=(-\sin\varphi,\cos\varphi)$ and $\mathbf{e}(\varphi)=(\cos\varphi,\sin\varphi)$ are a pair of orthonormal frame, $\rho$ is the radius of the curvature, and $C$ is the centre of curvature.

A point ( $x$, $y$ ) on a spiral is described by the following pair of equations:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \int_0^{\varphi} \rho(u) \begin{bmatrix} \cos u \\ \sin u \end{bmatrix} du \tag{1}$$

If the initial point of the spiral is $\mathbf{A}_0=(x_0, y_0)$ and $\rho=\rho_0+k\varphi$, then (1) becomes

$$\mathbf{A}(\varphi) = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \int_0^{\varphi} (\rho_0 + ku) \begin{bmatrix} \cos u \\ \sin u \end{bmatrix} du \tag{2}$$

where $k = ds/d\varphi$ determines the shape of a spiral. For a standard involute scroll with a constant wall thickness, $k=(r+d)/\pi$, where $r$ is the radius of the orbit and $d$ is the thickness of the wall [7].

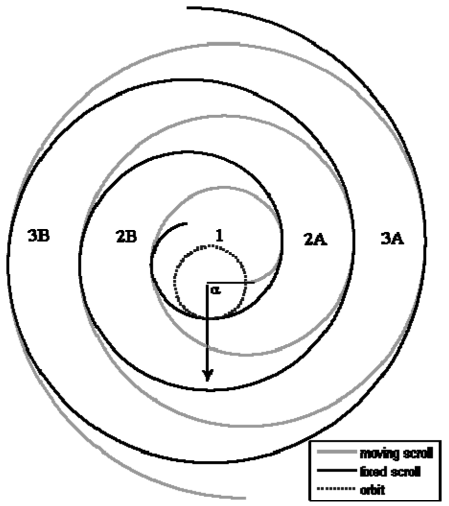


Fig. 3 A diagram of a simplified scroll-type air motor

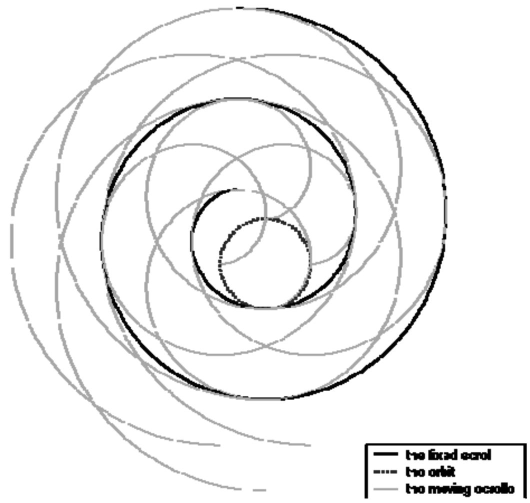

Fig. 4 A family of spirals

### B. *Relationship between the moving and the fixed scrolls*

To simplify analysis, in this paper, the thickness of scroll walls are not considered, that is, they have a zero thickness. Fig. 3 shows a simplified scroll air motor structure with three wraps of scrolls. In this classical design, both the moving scroll and the fixed scroll are circular involutes. The grey scroll is the moving scroll; the black one is the fixed scroll; the dashed circle in the centre is the orbit along which the moving scroll wobbles; $\alpha$ is the orbit angle that indicates the location of the moving scroll. The orbit angle varies at the period of $2\pi$. The chamber marked with "1" is the central chamber where the inlet is located. The chambers marked with "2A", "2B", "3A" and "3B" are two pairs of crescent side chambers that are sealed. Side chambers are located at the side of the central chamber. A scroll air motor, normally, have only one central chamber, but a certain pairs of side chambers.

When a scroll air motor is running, the moving scroll moves anticlockwise along the orbit. If we project some moments during one cycle onto one single figure, it is easy to say that the moving scroll forms a family of curve, and the fixed scroll is the envelope of the family as shown in Fig. 4.

The orbit is given as

$$\mathbf{D} = -r\mathbf{f} = r(\sin\alpha, \cos\alpha) \tag{3}$$

and the equation of the fixed scroll is represented by $\mathbf{A}(\varphi)$, then the family of the moving scroll is

$$\mathbf{A}(\varphi,\alpha) = \mathbf{A}(\varphi) + \mathbf{D}(\alpha) \tag{4}$$

From (3) and (4), the fixed scroll can be described by:

$$\mathbf{B}(\varphi) = \mathbf{A}(\varphi) + \mathbf{D}(\varphi + n\pi), \quad n \in Z \tag{5}$$

Because $\mathbf{D}(\varphi)$ is periodic with a period of $2\pi$, we have in fact two different envelops, one for each side of $\mathbf{A}(\varphi,\alpha)$, i.e.

$$\begin{aligned} \mathbf{B}_+(\varphi) &= \mathbf{A}_+(\varphi) + \mathbf{D}(\varphi) \\ \mathbf{B}_-(\varphi) &= \mathbf{A}_-(\varphi) - \mathbf{D}(\varphi) \end{aligned}. \tag{6}$$

### C. *Calculation of chamber volumes*

Green's Theorem gives the relationship between a line integral around a simple close curve and a double integral over the plane region bounded by a closed curve. It can be used to find areas. In Fig. 5, at the orbit angle $\alpha$, the moving scroll and the fixed scroll contact each other at some points. The outer side of the moving scroll, $\mathbf{A}_+$, contacts the inner side of the fixed scroll, $\mathbf{B}_+$, at $\varphi = 2n\pi$ $(n = 0,1,2,3\cdots)$. The inner side of the moving scroll, $\mathbf{A}_-$, contacts the outer side of the fixed scroll, $\mathbf{B}_-$, at $\varphi = 2n\pi$ $(n = 0,1,2,3\cdots)$. The subscriptions "+" and "-" indicate the two sides of a scroll. Because this paper presents a simplified model of a scroll air motor and the assumed wall thickness value is zero, $\mathbf{A}_-(\varphi) = \mathbf{A}_+(\varphi+\pi) = \mathbf{A}(\varphi+\pi)$ and $\mathbf{B}_-(\varphi) = \mathbf{B}_+(\varphi-\pi) = \mathbf{B}(\varphi-\pi)$. There is one central chamber and two side chambers (one pair) as shown in Fig. 5. The boundaries of the chambers have been parameterized as (4) and (5).

According to Green's Theorem, the volume of the central chamber is

$$\begin{aligned} V_c(\alpha) &= \frac{1}{2} z \int_{\alpha}^{\alpha+\pi} -y_{\mathbf{A}}(\varphi_{\mathbf{A}}) d(x_{\mathbf{A}}(\varphi_{\mathbf{A}})) + x_{\mathbf{A}}(\varphi_{\mathbf{A}}) d(y_{\mathbf{A}}(\varphi_{\mathbf{A}})) \\ &+ \frac{1}{2} z \int_{\alpha-\pi}^{\alpha} -y_{\mathbf{B}}(\varphi_{\mathbf{B}}) d(x_{\mathbf{B}}(\varphi_{\mathbf{B}})) + x_{\mathbf{B}}(\varphi_{\mathbf{B}}) d(y_{\mathbf{B}}(\varphi_{\mathbf{B}})) \end{aligned} \tag{6}$$

$Z$ stands for the height (or chamber thickness) of the scrolls.

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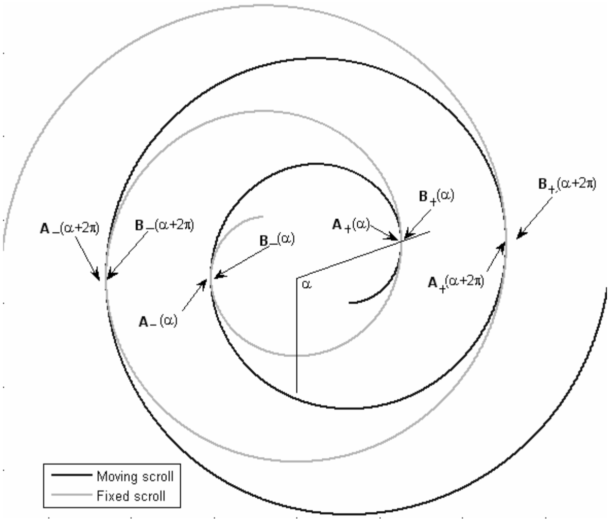


Fig. 5 The areas bounded by the scrolls

The volume of the side chamber bounded by $\mathbf{A}[\alpha, \alpha + 2\pi]$ and $\mathbf{B}[\alpha, \alpha + 2\pi]$ is calculated by

$$V_s(\alpha) = \frac{1}{2} z \int_{\alpha+2\pi}^{\alpha} - y_{\mathbf{A}}(\varphi_{\mathbf{A}}) d(x_{\mathbf{A}}(\varphi_{\mathbf{A}})) + x_{\mathbf{A}}(\varphi_{\mathbf{A}}) d(y_{\mathbf{A}}(\varphi_{\mathbf{A}})) + \frac{1}{2} z \int_{\alpha}^{\alpha+2\pi} - y_{\mathbf{B}}(\varphi_{\mathbf{B}}) d(x_{\mathbf{B}}(\varphi_{\mathbf{B}})) + x_{\mathbf{B}}(\varphi_{\mathbf{B}}) d(y_{\mathbf{B}}(\varphi_{\mathbf{B}})) \tag{7}$$

## IV. MOTION ANALYSIS

When a scroll air motor is running, every particle on the moving scroll is traveling along a circle, which is identical to the orbit. Thus, at time $t$, the direction of the motion, i.e. the direction of moving velocity, is the tangent direction of the point $\mathbf{D}(\alpha(t))$.

### *A. The driving segments and the balanced segments*

For a scroll-type air motor, some chambers are identical, so some segments of the scrolls are balanced by the pressure in the identical chambers. In Fig. 3, there are two pairs of identical chambers, "2A" and "2B", "3A" and "3B". The pressures in the identical chambers are of the same value. Thus, the segment between "2A" and "2B" is balanced. Furthermore, the pressure value in "3B" is lower than that in "2A", so the segment between "2A" and "3B" is a driving segment. For the same reason, the segment between "3A" and the atmosphere is also a driving segment.

### *B. Analysis of generalized force or torque*

Transient pressure in every chamber can be obtained. By integrating the pressures along the walls of the chambers individually, the driving force can be derived. Taking into consideration of frictions, stictions and the pay load, a mathematical description for the scroll driving force or torque is developed, which can represent the mechanical structure and the dynamical process of a scroll-type air motor.

In Fig. 6, the bigger arrows indicate the direction of motion of the moving scroll, the smaller arrows indicate the direction of the sum pressures at the points on the moving scroll. The sum force at a point is:

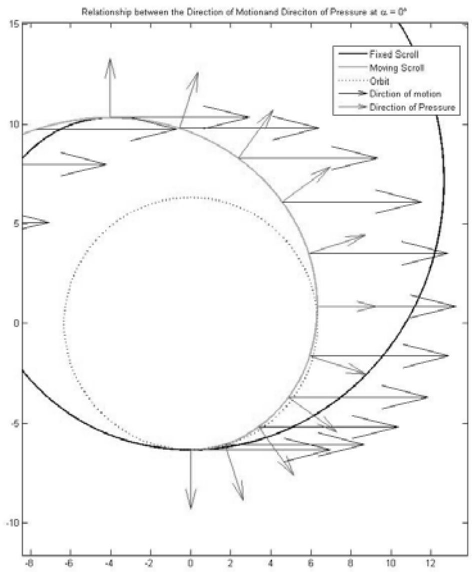


Fig. 6 Relationship between the direction of motion and the direction of the sum of the pressure

$$d\mathbf{F} = z\mathbf{p} ds \cos\theta \tag{8}$$

where $Z$ is the height of the scroll wall, $\mathbf{p}$ is the sum pressure at the point, $ds$ is the arc length, and $\theta = \varphi - \pi/2 - \alpha$ is the angle between the direction of movement and that of the pressure at the point. Thus the total generalized force in the direction of the motion to drive the motor moving is integral of (8).

## V. COMPLETE DYNAMIC PROCESS MATHEMATICAL MODEL OF A SCROLL-TYPE AIR MOTOR

### *A. State-space description*

To derive a mathematical model of a simplified scroll air motor, some assumptions are made as there is no air leakage of the motor, the compressed air is ideal gas, the static friction can be neglected, there is no heat transfer, motor surrounding temperature is constant, and the supply pressure is kept at a constant level. Let $x_1$ be orbit angle, $x_2$ angular velocity, $x_3$ pressure in the centre chamber, $x_4$ pressure in the first pair of side chambers, and $x_5$ pressure in the second pair of side chambers. Based on the above description, by applying Newton's second law of motion and the standard orifice theory [9], involving the mathematical model for control volumes, a state-space description of a scroll-type air motor with three-wrap scrolls can be derived. Equation (9) presents the state equations of a simplified scroll air motor with a proportional control valve.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{J}\left(\tau - M_f x_2\right)$$
$$\dot{x}_3 = -\frac{\dot{V}_c(x_1)}{V_c(x_1)}\gamma x_3 x_2 + \frac{1}{V_c(x_1)} R\gamma c_0 c_d c_k \sqrt{T} p_s X X_{\max} f\left(p_s / x_3\right) \tag{9}$$
$$\dot{x}_4 = -\frac{\dot{V}_s(x_1,1)}{V_s(x_1,1)}\gamma x_4 x_2$$
$$\dot{x}_5 = \begin{cases} -\frac{\dot{V}_s(x_1,2)}{V_s(x_1,2)}\gamma x_5 x_2 & \alpha \in [0,\pi] \\ p_{\text{atm}} & \alpha \in (\pi, 2\pi] \end{cases}$$

where

$$f(p_r) = \begin{cases} 1 & p_{\text{atm}} / p_u < p_r < c_r \\ \left(p_r^{2/\gamma} - p_r^{(1+\gamma)/\gamma}\right)^{1/2} & c_r \le p_r < 1 \end{cases}$$

$$\tau = \begin{cases} zr[(2\rho_0 + 2k\alpha + k\pi)(x_3 - x_4) \\ +(2\rho_0 + 2k\alpha + 5k\pi)(x_4 - x_5) \\ +(2\rho_0 + 2k\alpha + 9k\pi)(x_5 - p_{\text{atm}})] & \alpha \in [0,\pi] \\ zr[(2\rho_0 + 2k\alpha + k\pi)(x_3 - x_4) \\ +(2\rho_0 + 2k\alpha + 5k\pi)(x_4 - p_{\text{atm}})] & \alpha \in (\pi, 2\pi] \end{cases}$$

$J$ is the total inertia of the motor; $M_f$ is the friction coefficient; $V_c$ is the volume of the central chamber; $V_s$ is the volume of a side chamber; $\gamma = 1.4$ is the ratio of specific heats; $c_0 = 0.04$ ; $c_r = 0.528$ ; $c_k = 3.864$ ; $c_d = 0.8$ ; $X$ is the effective valve width; $X_{\max}$ is the maximal effective valve width; $p_s$ is the supply pressure; $p_{atm}$ is the pressure of atmosphere; $p_r = p_d / p_u$ ; $\tau$ stands for torque.

### B. *State shifting*

In order to solve the dynamic system (9), a method called state shifting must be introduced. A scroll air motor runs repeatedly in a period of $2\pi$. Compressed air goes into the scroll-type air motor. The first chamber it reaches is the central chamber. Along with motion of the air motor, at $x_1 = 2\pi$, a certain mass of compressed air is sealed in the first pair of side chambers. The initial value of $x_4$ is the final value of $x_3$. After another cycle, this certain mass of air is in the second pair of side chambers, so the initial value of $x_5$ is the final value of $x_4$ of the last cycle. The principle of the state shifting is that the state variation of a certain mass of compressed air is observed as it travels from the central chamber through the side chambers to the outlet. If there are $N$ pairs of side chambers, at every $x_1 = 2\pi$, the pressure of the $n^{th}$ chamber becomes the pressure of the $(n+1)^{th}$ chamber and then the air motor goes into another cycle [8].

## VI. ENERGY EFFICIENCY ANALYSIS

### A. *Energy carried by air*

Energy represents the potential for producing work that can be extracted from a substance. For compressed air, the potential is the available energy from its high pressure. It is the maximum work that can be extracted from air as it undergoes a reversible process form a given high pressure state to the atmospheric state on the surrounding of the atmosphere [1]. When the air temperature is equal to atmospheric temperature, the available energy per unit mass of air can be expressed by

$$e = \frac{pV}{m} \ln \frac{p}{p_{atm}}$$

The definition of air power is

$$P = GRT_{atm} \ln \frac{p}{p_{atm}} = pQ \ln \frac{p}{p_{atm}} \tag{10}$$

where $G$ stands for mass flow rate; $R$ is the gas constant; $Q$ stands for volumetric flow rate; $T_{atm}$ is atmospheric temperature. From (10), we know that if the air pressure $p$ is smaller or equal to the atmospheric pressure $p_{atm}$, the power $P$ is not greater than zero. That means air conveys available energy if and only if its pressure is greater than $p_{atm}$.

Air power consists of two parts. One is transmission power that represents pushing power from the upstream to the downstream. It is calculated by using the same expression as hydraulic power. Pneumatic cylinders normally utilise this part only. However, compressed air contains not only transmission power, but also, because of compressibility of air, expansion power. Even when the upstream is shut off, compressed air can still perform work by expanding. Thus another significant part of air power is expansion power that shows the work ability by air expansion [2].

Fig. 7 shows the percentage of expansion energy increases as the air pressure increases, while the potion of transmission energy decreases.

The temperature of air hardly influences air energy when we are discussing energy efficiency of a pneumatic actuator because the air power increases about 15% when the temperature difference is 100 K. Besides transmission energy and expansion energy, kinetic energy can also be converted into mechanical work. However it accounts for less than 5% of available energy when the average velocity is below 100 m/s at the pressure above $3 \times 10^5$ Pa [2]. Therefore, kinetic energy can normally be neglected when we are not analysing the internal energy distribution in pneumatic components.

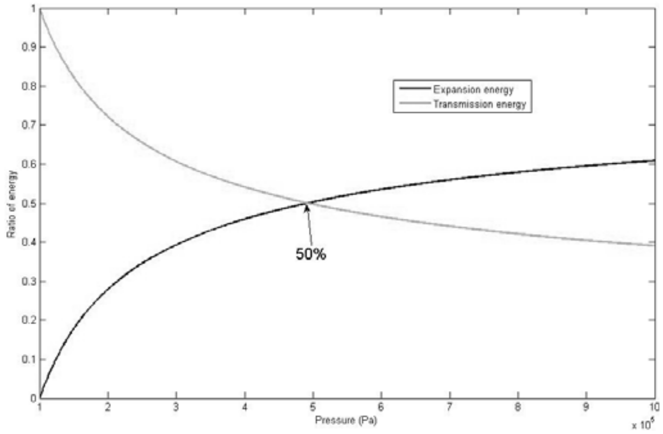


Fig. 7 Composition of air energy

### B. *Energy efficiency analysis of a scroll-type air motor*

Energy efficiency of a scroll-type air motor is calculated by:

$$\eta = \frac{\text{Power generated by the scroll air motor}}{\text{Air power}} \tag{11}$$

Power generated by scroll air motor is calculated by:

$$\text{Power (kw)} = \frac{\text{Torque (Nm)} \cdot \text{Speed (rpm)}}{9550} \tag{12}$$

Fig. 8 shows that, for the system (9), while the supply pressure increases from 0.35 MPa to 3 MPa, energy efficiency drops from 84% down to 46%.

The energy efficiency, i.e. ability of energy conversion, of the scroll-type air motor is much higher than the average energy efficiency of conventional pneumatic actuating systems, which is often lower than 20%. In fact, irreversible processes such as friction, heat transfer, and air mixture will cause air power loss, so the actual energy efficiency of a scroll-type air motor should below the theoretical value. In fact, energy efficiency of a scroll-type air motor mainly depends on supply air pressure, surrounding air pressure and ratio of expansion. Furthermore, high energy efficiency does not mean high power. However, it is still possible for a scroll-type air motor to have both high energy efficiency and high power. One can feed high pressure to a scroll-type air motor with a proper expansion ratio.

From the analysis above, a scroll-type air motor is able to convert more available energy of compressed air. In order to utilise air energy as much as possible, expansion ratio of side chambers is very important. Expansion ratio determines how much expansion energy is utilised. However the higher expansion ratio does not mean the higher energy efficiency. It must be ensure that the exhaust pressure is above atmosphere to avoid over expansion.

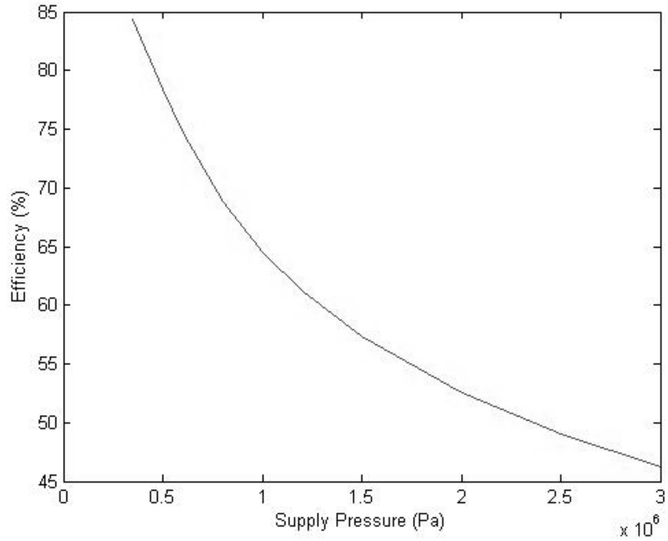


Fig. 8 Energy efficiency vs. supply pressure

## VII. Conclusion

This paper analysed the ability of a scroll-type air motor to convert air energy. The analysis is based on the simplified mathematical model of a scroll-type air motor derived in the paper. Available energy of compressed air was defined as a relative potential that only exists when the compressed air state differs from atmospheric surroundings. Air power consists of two parts: transmission power and expansion power. Normally, a pneumatic actuator like a cylinder uses only the transmission power. That leads to low energy efficiency of a pneumatic actuator. The analysis shows that a scroll-type air motor utilises both transmission power and expansion power. That results in high energy efficiency.

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