A Fixed Point Approach to Ulam Stability Problem for Cubic and Quartic Mappings in Non-Archimedean Fuzzy Normed Spaces

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Abstract— **A.Mirmostafaee and M.S.Moslehian[3] introduced the notion of non-Archimedean Fuzzy normed space in 2008. S.Y.Jang, J.R.Lee, C.Park and D.Y.Shin[17] proved the stability of quadratic functional equations in Fuzzy Banach space for a non-zero real number *a* with $a \neq \pm\frac{1}{2}$. The aim of present paper is to obtain some results for the stability of cubic and quartic mappings in non-Archimedean Fuzzy Normed Spaces.**

Index Terms—**Non-Archimedean, Fuzzy normed space, cubic and quartic mappings.**

1. INTRODUCTION

A classical question in the theory of functional equations is the following "When is it true that a function which approximately satisfies a functional equation ε must be close to an exact solution ε ?". If the problem accepts a solution, we say that the equation ε is stable. In 1940, S.M.Ulam [16] gave a wide ranging talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of important unsolved problems. Among those was the following question concerning the stability of homomorphisms; Let $(G_1,*)$ be a group and $(G_2,\circ,d)$ be a metric group with the metric *d*. Given $\varepsilon > 0$, does there exists a $\delta_\varepsilon > 0$ such that if a mapping $h: G_1 \to G_2$ satisfies the inequality

$$D(h(x*y), h(x)\circ h(y)) < \delta_\varepsilon \quad \forall\ x, y \in G_1,$$

then there is a mapping $H: G_1 \to G_2$ such that for each $x, y \in G_1 H(x*y) = H(x)\circ H(y)$ and $d(h(x), H(x)) < \varepsilon$?

In the next year, D.H.Hyers [7], gave answer to the above question for additive groups under the assumption that groups are Banach spaces.

Let E_1 be a normed space, E_2 a Banach space and suppose that the mapping $T: E_1 \to E_2$ satisfies the inequality

$$\|f(x+y) - f(x) - f(y)\| \le \varepsilon, \quad x, y \in E_1. \qquad (1)$$

Where $\varepsilon > 0$ is a constant. Then the limit $T(x) = \lim 2^{-n} f(2^n x)$ exists for each $x \in E_1$ and *T* is unique additive mapping satisfying

$$\|f(x) - T(x)\| \le \varepsilon \qquad x \in E_1.$$

Also, if for each *x* the function $t \to f(tx)$ from *R* to E_2 is continuous at a single point of E_1, then *T* is continuous everywhere in E_1, Moreover (1) is sharp.

In 1978, Th.M.Rassias[19] proved a generalization of Hyer's theorem for additive mapping as a special case.

Suppose that *E* and *F* are real normed spaces with *F* a complete normed space, $f: E \to F$ is a mapping such that for each fixed $x \in E$ the mapping $t \to f(tx)$ is continuous on *R*, and let there exist $\varepsilon \ge 0$ and $p \in [0,1)$

s.t. $\|f(x+y) - f(x) - f(y)\| \le \varepsilon(\|x\|^p + \|y\|^p)$, $x, y \in E$.

Then there exists a unique linear mapping $T: E \to F$

s.t. $\|f(x) - T(x)\| \le \varepsilon \dfrac{\|x\|^p}{(1-2^{p-1})} \qquad x \in E$

The case of the existence of unique additive mapping had been obtained by T.Aoki [18].

In 1994, P. Gauruta [14] provided a further generalization of Th.M.Rassias' theorem in which he replaced the bound $\varepsilon(\|x\|^p + \|y\|^p)$ by a general control function $\phi(x, y)$ for the existence of a unique linear mapping. The functional equation $f(2x+y) + f(2x-y) = 2f(x+y) + 2f(x-y) + 12f(x)$ is said to be the cubic functional equation since cx^3 is its solution. Every solution of the cubic functional equation is said to be cubic mapping. The stability problem for the cubic functional equation was proved by Jun and Kim[12] for mappings $f: X \to Y$ where *X* is a real normed space and *Y* is Banach space. The functional equation $f(2x + y) + f(2x - y) = 4f(x + y) + 4f(x - y) + 24f(x) - 6f(y)$ is said to be a quartic functional equation. The stability problem for the quartic functional equation was proved by J.M.Rassias [10] for mappings $f: X \to Y$ where *X* is a real normed space and *Y* is Banach space. In the present paper, we obtain some results regarding stabilility of cubic and quartic mappings in non-Archimedean Fuzzy Normed Spaces.

During the last three decades, the theory of non-Archimedean spaces has gained the interest of physicists for their research, in particular the problems that emerge in quantum physics, p-adic strings and superstrings. Although many results in the classical normal space theory have a non-Archimedean counterpart, their proofs are different and require a rather new kind of intuition. It may be noted that $|n| \le 1$ in each valuation field every triangle is isosceles and there may be no unit vector in a non-Archimedean

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framework is of the special interest.

In the present paper we obtain some results regarding stability of cubic and quartic mappings in non-Archimedean Fuzzy Normed spaces.

2. PRELIMINIARIES

Definition 2.1. Let K be a field. A non-Archimedean absolute value on K is a function $|\cdot|: K \to R$ such that for any $a,b \in K$ we have

(i) $|a| \geq 0$ and equality holds if and only if $a = 0$,

(ii) $|ab| = |a||b|$

(iii) $|a+b| \leq \max\{|a|,|b|\}$

The condition (iii) is called the strict triangle inequality. By (ii), we have $|1| = |-1| = 1$. Thus, by induction, it follows from (iii) that $|n| \leq 1$ for each integer n. We always assume in addition that $|\cdot|$ is non trivial, i.e. that there is an $a_0 \in K$ such that

$|a_0| \notin \{0,1\}$

Definition 2.2. Let X be a linear space over a non-Archimedean field K. A function $N : X \times R \to [0,1]$ is said to be a non-Archimedean fuzzy norm on X if for all $x, y \in X$ and all $x, t \in K$.

($N1$) $N(x,c) = 0$ for $c \leq 0$;

($N2$) $x = 0$ if and only if $N(x, c) = 1$ for all $c > 0$;

($N3$) $N(cx,t) = N\left(x, \frac{t}{|c|}\right)$ if $c \neq 0$;

($NA4$) $N(x+y, \max\{s,t\}) \geq \min\{N(x,s), N(y,t)\}$

($N5$) $\lim_{t\to\infty} N(x,t) = 1$.

The pair (X, N) is called a non-Archimedean fuzzy normed space. Clearly, if ($NA4$) holds then so is

(N4) $N(x + y, s + t) \geq \min\{N(x, s), N(y, t)\}$.

A classical vector space over a complex or real field satisfying ($N1$) – ($N5$) is called fuzzy normed space. It is easy to see that ($NA4$) is equivalent to the following condition

($NA4'$) $N(x + y, t) \geq \min\{N(x, t), N(y, t)\}$ $(x, y \in X;\ t \in R)$.

Example 2.3. Let (X, $\|\cdot\|$) be a non-Archimedean normed linear space. Then

$$N(x,t) = \begin{cases} \frac{t}{t+\|x\|} & t > 0, x \in X \\ 0 & t \leq 0, x \in X \end{cases}$$

is a non-Archimedean fuzzy norm on X.

Example 2.4. Let (X, $\|\cdot\|$) be a non-Archimedean linear space. Then

$$N(x,t) = \begin{cases} 0, & t \leq \|x\| \\ 1, & t > \|x\| \end{cases}$$

is a non-Archimedean fuzzy norm on X.

Definition 2.5. Let (X, N) be a non-Archimedean fuzzy normed linear space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x \in X$ such that

$$\lim_{n\to\infty} N(x_n - x, t) = 1 \quad \text{for all} \quad t > 0$$

In that case, x is called the limit of the sequence $\{x_n\}$ and we denote it by N-lim $x_n = x$.

A sequence $\{x_n\}$ in X is called Cauchy if for each $\varepsilon > 0$ and each $t > 0$. There exists n_0 such that for all $n \geq n_0$ and all $p > 0$ we have $N(x_{n+p} - x_n, t) > 1 - \varepsilon$. Due to

$N(x_{n+p} - x_n, t) \geq \min\{N(x_{n+p} - x_{n+p-1}, t), \ldots, N(x_{n+1} - x_n, t)\}$

the sequence $\{x_n\}$ is Cauchy if for each $\varepsilon > 0$ and each $t > 0.$, there exists n_0 such that for all $n \geq n_0$ we have

$N(x_{n+1} - x_n, t) > 1 - \varepsilon$

It is easy to see that every convergent sequence in a non-Archimedean fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then non-Archimedean fuzzy normed space is called a non-Archimdean fuzzy Banach space.

3. MAIN RESULTS

In the rest of this paper, unless otherwise explicitly stated, we will assume that K is a non-Archimedean field, X is a vector space over K, (Y, N) is a non-Archimedean fuzzy Banach space over K and (Z, N') is a (Archimedean or non-Archimedean) fuzzy normed space.

In this paper, we first establish the stability of the cubic functional equations in non-Archimedean fuzzy normed space.

Theorem 3.1. Let X be a linear space, (Z, N') be non-archimedean fuzzy normed space, and $\phi : X \times X \to Z$ be a function such that for some $0 < \alpha < 8$

$$N'(\phi(2x,0),t) \geq N'(\alpha\phi(x,0),t) \forall x \in X, t > 0\ f(0) = 0 \qquad (3.1)$$

and $\lim_{n\to\infty} N'(\phi(2^n x, 2^n y), 8^n t) = 1$ for all $x, y \in X$ and all $t > 0$. Let (Y, N) be complete non-Archimedean fuzzy space. If $f : X \to Y$ is a mapping such that

$$N(f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 12f(x), t) \geq N'(\phi(x,y),t), \qquad (3.2)$$

then there exists a unique cubic mapping $C : X \to Y$ such that

$$N(f(x) - C(x), t) \geq N'(\phi(x,0), 2(8-\alpha)t) \qquad (3.3)$$

Proof. From (3.2), it follows that

$$\inf\{t > 0;\ N(f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 12f(x), t) > k\} \qquad (3.4)$$

$\leq \inf\{t > 0;\ N'(\phi(x, y), t) > 1 - \lambda\}$. $\forall\ x, y \in X$, $\lambda \in (0, 1)$

Putting $y = 0$ in (3.4), we get

$$\inf\left\{t > 0; N\left(\frac{f(2x)}{8} - f(x), t\right) > k\right\} \leq \frac{1}{16} \qquad (3.5)$$

$$\inf\{t > 0; B'(\phi(x,0),t) \qquad \forall x \in X$$

Replacing x by $2^n x$ in (3.5) and using (3.1), we obtain

$$\inf\left\{t > 0; N\left(\frac{f(2^{n+1}x)}{8^{n+1}} - \frac{f(2^n x)}{8^n} f(x), 8^n t\right) > k\right\} \leq \frac{1}{16}$$

$$\inf\{t > 0; N'(\phi(2^n x, 0), t)\}$$

$$\inf\left\{t > 0; N\left(\frac{f(2^{n+1}x)}{8^{n+1}} - \frac{f(2^n x)}{8^n} f(x), t\right) > k\right\} \leq \frac{1}{16 \times 8^n}$$

$$\inf\{t > 0; N'(\alpha^n \phi(x,0), t)\}$$

$$\inf\left\{t > 0; N\left(\frac{f(2^{n+1}x)}{8^{n+1}} - \frac{f(2^n x)}{8^n} f(x), t\right) > k\right\} \leq \frac{\alpha^n}{16 \times 8^n}$$

$$\inf\{t > 0; N'(\phi(x,0), t)\}$$

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(3.6)

It follows from

$$(f(2^n x)/8^n) - f(x) = \sum_{k=0}^{n-1}\left(\frac{f(2^{k+1}x)}{8^{k-1}} - \frac{f(2^k x)}{8^k}\right)$$

and (3.6) using (NA4') that

$$\inf\left\{t > 0; N\left(\frac{f(2^n x)}{8^n} - f(x), t\right) > k\right\}$$

$$= \inf\{t > 0; N\left(\sum_{k=0}^{n-1}\left(\frac{f(2^{k+1}x)}{8^{k-1}} - \frac{f(2^k x)}{8^k}, t\right) > k\right)$$

$$\leq \sum_{k=0}^{n-1}\inf\left\{t > 0; N\left(\frac{f(2^{k+1}x)}{8^{k+1}} - \frac{f(2^k x)}{8^k}, t\right) > k\right\}$$

$$\leq \sum_{k=0}^{n-1}\frac{1}{16\times 8^k}\inf\{t > 0; N'(\phi(2^k x, 0), t) > k\}$$

$$\leq \sum_{k=0}^{n-1}\frac{\alpha^k}{16\times 8^k}\inf\{t > 0; N'((\phi(x,0),t) > k\} \qquad (3.7)$$

Replacing x with $2^m x$ in (3.7), we have

$$\inf\{t > 0; N\left(\left(\frac{f(2^{n+m}x)}{8^{n+m}} - \frac{f(2^m x)}{8^m}, t\right) > k\right)$$

$$\leq \sum_{k=0}^{n-1}\frac{\alpha^k}{16\times 8^{k+m}}\inf\{t > 0; N'((\phi(2^m x, 0), t) > k\}$$

$$\leq \sum_{k=0}^{n-1}\frac{\alpha^{k+m}}{16\times 8^{k+m}}\inf\{t > 0; N'((\phi(x,0),t) > k\}$$

$$\leq \sum_{k=m}^{m+n-1}\frac{\alpha^k}{16\times 8^k}\inf\{t > 0; N'((\phi(x,0),t) > k\}$$

$$= \frac{1}{16}\inf\{t > 0; N'((\phi(x,0),t) > k\}\sum_{k=m}^{m+n-1}\left(\frac{\alpha}{8}\right)^k \qquad (3.8)$$

Then $\left\{\frac{f(2^n x)}{8^n}\right\}$ is a Cauchy sequence in (Y, N). Since (Y, N) is a complete non-Archimedean fuzzy normed space, this sequence converges to some point $C(x) \in Y$. Fix $x \in X$ and put $m = 0$ in (3.8). Then we obtain

$$\inf\left\{t > 0; N\left(\frac{f(2^n x)}{8^n}\right) - f(x)), t) > k\right\}$$

$$\leq \frac{1}{16}\inf\{t > 0; N'((\phi(x,0),t) > k\}\sum_{k=0}^{n-1}\left(\frac{\alpha}{8}\right)^k \qquad (3.9)$$

and so

$\inf\{t > 0; N(C(x) - f(x), t) > k\}$

$$\leq \inf\left\{t > 0; N\left(C(x) - \frac{f(2^n x)}{8^n}, t\right) > k\right\}$$

$$+ \inf\left\{t > 0; N\left(\frac{f(2^n x)}{8^n} - f(x), t\right) > k\right\}$$

$$\leq \inf\left\{t > 0; N\left(C(x) - \frac{f(2^n x)}{8^n}, t\right) > k\right\}$$

$$+ \frac{1}{16}\inf\{t > 0; N'((\phi(x,0),t) > k\}\sum_{k=0}^{n-1}\left(\frac{\alpha}{8}\right)^k \qquad (3.10)$$

Taking the limit as $n \to \infty$ and using (3.10), we get

$\inf\{t > 0; N(C(x) - f(x), t) > k\}$

$$\leq \frac{1}{16 - 2\alpha}\inf\{t > 0; N'(\phi(x,0), t) > k\} \qquad (3.11)$$

i.e.

$\inf\{t > 0; N(C(x) - f(x), t) > k\}$

$\leq \inf\{t > 0; N'(\phi(x, 0), 2t(8 - \alpha)) > k\}$ (3.12)

$N(C(x) - f(x), t) \geq N'(\phi(x, 0), 2t(8 - \alpha))$ (3.13)

then replacing x and y by $2^n x$ and $2^n y$ in (3.2), we get

$$N\left(\frac{f(2^n(2x+y))}{8^n} + \frac{f(2^n(2x-y))}{8^n} - \frac{2f(2^n(x+y))}{8^n}\right.$$

$$\left.\frac{2f(2^n(x-y))}{8^n} - \frac{12f(2^n(x))}{8^n}, t\right)$$

$$\geq N'(\phi(2^n x, 2^n y), 8^n t), \forall x, y \in X, t > 0. \qquad (3.14)$$

Since $\lim_{n\to\infty} N' f(2^n x, 2^n y), 8^n t) = 1.$ Thus C satisfies

$f(2x + y) + f(2x - y) = 2f(x + y) + 2f(x - y) + 12 f(x)$

To prove the uniqueness of cubic mapping C assume that there exists a cubic mapping $D : X \to Y$ which satisfies (3.3). Fix $x \in X$. Clearly

$$C(2^n x) = 8^n(C(x)) \text{ and } D(2^n x) = 8^n D(x) \quad \forall\ n \in N$$

From (3.3), using (NA4')

$$N(C(x) - D(x), t) = \lim_{n\to\infty} N\left(\frac{C(2^n x)}{8^n} - \frac{D(2^n x)}{8^n}, t\right)$$

$$N\left(\frac{C(2^n x)}{8^n} - \frac{D(2^n x)}{8^n}, t\right) \geq \min$$

$$\geq N'(\phi(2^n x, 0), 8^n 2(8-\alpha)t)$$

$$\geq N\left(\phi(x,0), \frac{8^n 2(8-\alpha)t}{\alpha^n}\right) \qquad (3.15)$$

Since

$$\lim_{n\to\infty}\left(\frac{8^n 2(8-\alpha)t}{\alpha^n}\right) = \infty,$$

we get

$$\lim_{n\to\infty} N'\left(\phi(x,0), \frac{8^n 2(8-\alpha)t}{\alpha^n}\right) = 1.$$

Therefore it implies

$N(C(x) - D(x), t) = 1$ for all $t > 0$

and so $C(x) = D(x)$.

Corollary 3.2. Let X be a linear space (Z, N'), be a non-Archimedean fuzzy normed linear space, (Y, N) be complete non-Archimedean fuzzy normed space. Let p, q be non-negative real numbers and let $z_0 \in Z$. If $f : X \to Y$ is a mapping such that

$N(f(2x + y) + f(2x - y) - 2f(x + y) - 2f(x - y) - 12f(x), t)$

$\geq N'((\|x\|^p + \|y\|^q)z_0, t)\ \forall\ x, y \in X, t > 0$ (3.16)

$f(0) = 0$ and $p, q < 3$, then there exists a unique cubic mapping $C : X \to Y$ such that

$N(f(x) - C(x), t) \geq N'(\|x\|^p z_0, 2(8 - 2^p)t)\ \ \forall\ x \in X, t > 0$ (3.17)

Proof. Let $\phi\ X \times X \to Z$ be defined by

$\phi(x, y) = (\|x\|^p + \|y\|^q)\, z_0$ and $\alpha = 2^p$

then using theorem (3.1) proof is obtained.

Corollary 3.3. Let X be a linear space (Z, N') be a non-Archimedean fuzzy normed linear space and (Y, N) a complete non-Archimedean fuzzy normed linear space. Let

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$z_0 \in Z$. If $f : X \to Y$ is a mapping such that

$$N(f(2x + y) + f(2x - y) - 2f(x + y) - 2f(x - y) - 12f(x), t) \geq N'(\in z_0, t), \ \forall x, y \in X, t > 0 \quad (3.18)$$

and $f(0) = 0$, then there exists a unique cubic mapping $C : X \to Y$ such that

$$N(f(x) - C(x), t) \geq N'(\in z_0, 14t) \ \forall x \in X, t > 0 \quad (3.19)$$

Proof. Let $\phi : X \times X \to Z$ be defined by $\phi(x, y) = \in z_0$. Then, the proof follows from theorem (3.1) by $\alpha = 1$.

Now, we establish the stability of the quartic functional equations in non-Archimedean fuzzy normed space.

Theorem 3.4. Let X be a linear space (Z, N') a non-Archimedean fuzzy normed linear space and $\phi : X \times X \to Z$ a function such that for some $0 < \alpha < 16$,

$$N'(\phi(2x, 0), t) \geq N'(\alpha\, \phi(x, 0), t) \ \ \forall x \in X, t > 0 \ f(0) = 0$$

and

$$\lim_{n\to\infty} N'(\phi(2^n x, 2^n y)), 16^n t) = 1 \text{ for all } x, y \in X \quad (3.20)$$

and all $t > 0$. Let (Y, N) be a complete non-Archimedean fuzzy normed linear space. If $f : X \to Y$ is a mapping such that

$$N(f(2x + y) + f(2x - y) - 4 f(x + y) - 4f(x - y) - 24 f(x) + 6f(y), t) \geq N'(\phi(x, y), t) \ \forall x, y \in X, t > 0 \quad (3.21)$$

then there exists a unique quadratic mapping Q : X $\to$ Y such that

$$N(f(x) - Q(x), t) \geq N'(\phi(x, 0), 2(16 - \alpha)\, t) \quad (3.22)$$

Proof. The proof of this theorem follows as the proof of theorem (3.1)

Corollary 3.5 : Let X be a linear space (Z, N'), be a non-Archimedean fuzzy normed linear space, (Y, N) be complete non-Archimedean fuzzy normed space. Let p, q be non-negative real numbers and let $z_0 \in Z$. If $f : X \to Y$ is a mapping such that

$$N(f(2x + y) + f(2x - y) - 4 f(x + y) - 4f(x - y) - 24 f(x) + 6f(y), t) \geq N'((\|x\|^p + \|y\|^q) z_0, t) \ \forall x, y \in X, t > 0$$

$f(0) = 0$ and p, $q < 4$, then there exists a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq N'(\|x\|^p z_0, 2(16 - 2^p)t) \ \ \forall x \in X, t > 0$$

Proof. Let ϕ X $\times$ X $\to$ Z be defined by

$$\phi(x, y) = (\|x\|^p + \|y\|^q)\, z_0 \text{ and } \alpha = 2^p$$

then using theorem (3.4) proof is obtained.

Corollary 3.6. Let X be a linear space (Z, N') be a non-Archimedean fuzzy normed linear space and (Y, N) a complete non-Archimedean fuzzy normed linear space. Let $z_0 \in Z$. If $f : X \to Y$ is a mapping such that

$$N(f(2x + y) + f(2x - y) - 4 f(x + y) - 4f(x - y) - 24 f(x) + 6f(y), t) \geq N'(\in z_0, t), \ \forall x, y \in X, t > 0$$

and $f(0) = 0$, then there exists a unique quartic mapping $Q : X \to Y$ such that

$$N(f(x) - Q(x), t) \geq N'(\in z_0, 30t) \ \forall x \in X, t > 0$$

Proof. Let $\phi : X \times X \to Z$ be defined by $\phi(x, y) = \in z_0$. Then, the proof follows from theorem (3.4) by $\alpha = 1$.

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