Deterministic and Probabilistic EOQ models for products having Power Demand Pattern

Singh Sarbjit, Singh Shivraj

Abstract-In this paper models for deteriorating items having deterministic and probabilistic power demand pattern with variable rate of deterioration are developed. To make these models suited to present environment, effect of inflation is also investigated. The concept of permissible delay has also been incorporated. The total cost of inventory is obtained first, then an optimal order quantity and maximum allowable shortage is obtained. The purpose of this research is to aid the retailers in economically stocking the inventory under the influence of different decision criteria such as time value of money, inflation, power demand pattern and variable deterioration rate.

Key words: Inflation, Delayed payment, Power demand pattern, Shortages.

I.INTRODUCTION

Most of the earlier inventory models consider that demand rate is constant. This is a feature of static environment, while in today's dynamic environment most of the things are not constant. In real life situations, generally demand for items increases with time. Most of the companies are working towards increasing demand of their items with time.

Inventory problems involving time variable demand patterns have received attention from several researchers. This type of problem was first discussed by Stanley and Sivazlian (1973). But there is no specific assumption concerning demand in the problem. Next, Silver and Meal (1973) established an approximate solution technique of a deterministic inventory model with time-dependent demand. Donaldson (1977) developed an optimal algorithm for solving classical no-shortage inventory model analytically with linear trend in demand over fixed time horizon. Ritchie (1984) gave a simple optimal solution for the EOQ with linear increasing demand. Dutta and Pal (1987) considered both the deterministic and probabilistic versions of power demand pattern with variable rate of deterioration. U. Dave (1989) proposed a deterministic lot-size inventory model with shortages and a linear trend in demand. Goswami and Chaudhuri (1991) discussed different types of inventory models with linear trend in demand. Hariga (1995) studied the effects of inflation and time value of money on an inventory model with time-dependent demand rate and shortages Bhunia et al (1998) developed an inventory model of deteriorating items with lot-size dependent replenishment cost and linear trend in demand.

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Bhunia et al (2001) extended there study to a deterministic inventory model with two levels of storage, a linear demand trend for a fixed time horizon Khanra and Chaudhuri (2003) discussed an order level decaying inventory model with such time dependent quadratic demand. Zhou et al (2004) gave note on an inventory model for deteriorating items with stock dependent and time varying demand rate. Sarbjit and Shivraj (2007) developed an inventory model for items having linear demand pattern under the effect of permissible delay and inflation.

In this study we have considered the concept of permissible delay in payments under the effect of inflation and time value of money with time dependent demand rate. We have considered two cases with respect to depletion time T_1 which is either greater than or less than permissible delay. Thus this study will help inventory managers in deciding their stock of inventory having time dependent demand

II. NOTATIONS

f	Inflation rate.
i	Inventory carrying rate.
S	Period with shortage
T	Length of inventory cycle, time units
θ	Rate of deterioration per unit time
i_p	$I_p - r$
r	Discount rate representing the time value of money
I_p	Nominal interest paid per Dollar per unit time
i_e	$I_e - r$
I_e	Nominal interest at time t=0
$I_e(t)$	Rate of interest earned at time t Dollars per Dollar per unit time.
$I_p(t)$	Interest rate paid at time t Dollars per Dollar per unit time.
I_T^i	Total interest earned per cycle with inflation
P_T^i	Interest payable per cycle under inflation.
R	$f - r =$ Present value of the nominal inflation.
Q	Order quantity, units/order.
c	Unit cost of per item at time $t = 0$, Dollars/unit
c_b	Backorder cost at time $t = 0$, Dollars/unit
c_D^i	Total cost of deterioration per cycle with

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

inflation

c_H^i Total holding cost per cycle with inflation

III. Mathematical Model and Analysis for Deterministic Case

In this model power demand pattern is considered with variable rate of deterioration. Depletion of the inventory occurs due to demand (Supply) as well as due to deterioration which occurs only when there is inventory i.e., during the period $[0, T_1]$. For this period the inventory at any time t is given by

$$\frac{dI(t)}{dt}+\theta tI(t)=\frac{-dt^{(1-n)/n}}{nT^{1/n}} \quad 0\le t\le T_1 \qquad (1)$$

$$I(t).e^{\frac{\theta t^2}{2}}=\frac{-d}{T^{1/n}}\left[t^{1/n}+\frac{\theta}{2}\frac{t^{1+2n/n}}{(2n+1)}\right]+C_1$$

Where at t = 0, I (t) = I_0. Putting this value in the above equation, we get

$I_0 = C_1$ This gives

$$I(t)=I_0e^{\frac{-\theta t^2}{2}}-\frac{de^{\frac{-\theta t^2}{2}}}{T^{1/n}}\left[t^{1/n}+\frac{\theta}{2}\frac{t^{1+2n/n}}{(1+2n)}\right] \qquad (2)$$

It is obvious that at t = T_1, I (T_1) = 0. So equation 2 yields

$$I_0=\frac{d}{T^{1/n}}\left[T_1^{1/n}+\frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right]=Q-b \qquad (3)$$

Here b being the maximum backorder (Shortage) level permitted in each cycle.

Substituting the above value of I_0 in equation (2), we get

$$I(t)=\frac{de^{\frac{-\theta t^2}{2}}}{T^{1/n}}\left[\left(T_1^{1/n}-t^{1/n}\right)+\frac{\theta}{2}\frac{\left(T_1^{1+2n/n}-t^{1+2n/n}\right)}{(1+2n)}\right]$$

$$0\le t\le T_1 \qquad (4)$$

And I(t) = 0 when, $T_1 \le t \le T$. The total demand during T_1 is $\int_0^{T_1}\frac{dt^{\frac{1}{n}-1}}{nT^{1/n}}dt$. The amount of items which deteriorates during one cycle is given by

$$I_0=\frac{d}{T^{1/n}}\left[T_1^{1/n}+\frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right]=Q-b$$

$$D_T=I_o-\int_0^{T_1}\frac{dt^{\frac{1}{n}-1}}{nT^{1/n}}dt=\frac{d}{T^{1/n}}\left[\frac{\theta T_1^{\frac{2n+1}{n}}}{2(2n+1)}\right] \qquad (5)$$

The Inflation Model

There are two distinct cases in this type of inventory system

I. Payment at or before the total depletion of inventory($M \le T_1 < T$)

II. After depletion payment($T_1 < M$)

Case I (i.e., payment at or before the total depletion of inventory)

(a) Since the ordering is made at time t=0, the inflation does not affect the ordering cost. Thus the ordering cost for items is fixed at A Dollars/order

(b) Since $I_0=\frac{d}{T^{1/n}}\left[T_1^{1/n}+\frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right]$, the value of this inventory at time t=0, is c I_0. The present value of the items sold is $\int_0^{T_1}\frac{c_0dt^{\frac{1}{n}-1}}{nT^{1/n}}dt$. Hence the cost of deterioration per cycle time T under inflation, C_D^i is given by

$$C_D^i=cI_0-\int_0^{T_1}\frac{c_0dt^{\frac{1}{n}-1}}{nT^{1/n}}dt=\frac{cd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)}-\frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right] \qquad (6)$$

(c) The holding cost under inflation is given by

$$C_H^i=i\int_0^{T_1}c_0I(t)dt$$

$$=\frac{icd}{T^{1/n}}\left[\begin{array}{l}\left(T_1+\frac{RT_1^2}{2}-\frac{\theta T_1^3}{6}-\frac{R\theta T_1^4}{8}\right)T_1^{1/n}-\\ \left(\frac{nT_1^{\frac{1+n}{n}}}{n+1}+\frac{nRT_1^{\frac{1+2n}{n}}}{2n+1}-\frac{\theta nT_1^{\frac{1+3n}{n}}}{3n+1}-\frac{R\theta nT_1^{\frac{1+4n}{n}}}{4n+1}\right)\end{array}\right]$$

$$+\frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}}+\frac{RT_1^{\frac{1+4n}{n}}}{2}-\frac{\theta T_1^{\frac{1+5n}{n}}}{6}-\frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right)$$

$$-\frac{\theta}{2(1+2n)}\left(\frac{nT_1^{\frac{1+3n}{n}}}{3n+1}+\frac{nRT_1^{\frac{1+4n}{n}}}{4n+1}-\frac{\theta nT_1^{\frac{1+5n}{n}}}{5n+1}-\frac{R\theta nT_1^{\frac{1+6n}{n}}}{6n+1}\right) \qquad (7)$$

(d) The interest payable rate at time t is $e^{I_pt}-1$ dollars per dollar, so the present value (at t=0) of interest payable rate at time t is $I_p(t)=\left(e^{I_pt}-1\right)e^{-rt}$ rupees per rupees. Therefore the interest payable per cycle for the inventory not sold after the due date M is given by

$P_T^i=\int_M^T cI_p(t)I(t)dt \quad =\int_M^{T_1}cI_p(t)I(t)dt$ as I (t) = 0 for $T_1 \le t \le T$

$$=\frac{cd}{T^{1/n}}\left[\frac{(i_p-r)T_1^{1/n}(T^2-M^2)}{2}\left\{1-\frac{\theta}{4}\left(T_1^2+M^2\right)+\frac{\theta T_1^2}{1+2n}\right\}-(i_p-r)n\right.$$

$$\left.-(i_p-r)n\left\{\frac{\left(T_1^{\frac{2n+1}{n}}-M^{\frac{2n+1}{n}}\right)}{2n+1}-\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{4n+1}+\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)}\right\}\right] \qquad (8)$$

Where $i_p = I_p - r$

Similarly, the present value of the interest earned at time t, $I_e(t)$ is $\left(e^{I_et}-1\right)e^{-rt}$. Considering inflated unit cost at time t as $c_t = ce^{Rt}$, the interest earned per cycle, I_T^i is the interest earned up to time T_1 and it is given by

$$I_T^i=\int_0^T c_oI_e(t)td\frac{t^{\frac{1}{n}-1}}{nT^{1/n}}dt=\frac{cd}{T^{1/n}}\left[(i_e-r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1}+(i_e+r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right] \qquad (9)$$

The backorder cost per cycle under inflation, C_B^i is given by

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$C_B^i = \int_0^{T-T_1} c_b e^{R(T_1+t)} \frac{dt^{\frac{1}{n}-1}}{nT^{1/n}} dt$$

$$= \frac{c_b d}{T^{1/n}} (T - T_1)^{\frac{1}{n}} (1 + RT) \qquad (10)$$

Since the backorder starts at $t=T_1$

The Variable Cost Function

The total variable cost per cycle C_{VT}, is defined as

$$C_{VT}(T_1, T) = A + C_D^i - I_T^i + C_B^i + C_H^i + P_T^i \qquad (11)$$

Substituting the values from equation 6-10 in equation 11, we have C_{VT} in terms of T_1 and T

$$C_{VT}(T_1, T) = A + \frac{cd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)} - \frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right] -$$

$$\frac{cd}{T^{1/n}}\left[(i_e - r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1} + (i_e + r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right]$$

$$+ \frac{c_b d}{T^{1/n}}(T - T_1)^{\frac{1}{n}}(1 + RT)$$

$$\frac{icd}{T^{1/n}}\left[\begin{array}{l} \left(T_1 + \frac{RT_1^2}{2} - \frac{\theta T_1^3}{6} - \frac{R\theta T_1^4}{8}\right)T_1^{1/n} - \\ \left(\frac{nT_1^{\frac{1+n}{n}}}{n+1} + \frac{nRT_1^{\frac{1+2n}{n}}}{2n+1} - \frac{\theta n T_1^{\frac{1+3n}{n}}}{3n+1} - \frac{R\theta n T_1^{\frac{1+4n}{n}}}{4n+1}\right) \\ + \frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}} + \frac{RT_1^{\frac{1+4n}{n}}}{2} - \frac{\theta T_1^{\frac{1+5n}{n}}}{6} - \frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right) \\ - \frac{\theta}{2(1+2n)}\left(\frac{nT_1^{\frac{1+3n}{n}}}{3n+1} + \frac{nRT_1^{\frac{1+4n}{n}}}{4n+1} - \frac{\theta n T_1^{\frac{1+5n}{n}}}{5n+1} - \frac{R\theta n T_1^{\frac{1+6n}{n}}}{6n+1}\right) \end{array}\right]$$

$$+$$

$$\frac{cd}{T^{1/n}}\left[\begin{array}{l} \frac{(i_p - r)T_1^{1/n}(T^2 - M^2)}{2}\left\{1 - \frac{\theta}{4}\left(T_1^2 + M^2\right) + \frac{\theta T_1^2}{1+2n}\right\} - \\ (i_p - r)n - (i_p - r)n\left\{\begin{array}{l} \frac{\left(T_1^{\frac{2n+1}{n}} - M^{\frac{2n+1}{n}}\right)}{2n+1} - \\ \frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}} - M^{\frac{4n+1}{n}}\right)}{4n+1} + \frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}} - M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)} \end{array}\right\} \end{array}\right] \qquad (12)$$

Multiple Inventories Cycle per Year

The inflation and time value of money exist for each cycle of replenishment, so we need to consider the effect over the time horizon NT. Let there be N complete cycle during a year. Hence, NT=1. The total cost during total time is given by

$$C_T = C_{VT} \times \left[1 + e^{2RT} + e^{3RT} + \dots\dots\dots + e^{(N-1)RT}\right]$$

$$= C_{VT} \times \left[\frac{1 - e^{NRT}}{1 - e^{RT}}\right] = C_{VT} \times \left[\frac{1 - e^{R}}{1 - e^{RT}}\right] \qquad (13)$$

The value of T and T_1 which minimize C_T may be obtained by simultaneously solving

$$\frac{\partial C_T}{\partial T}(T_1, T) = 0 \quad \text{And} \quad \frac{\partial C_T}{\partial T_1}(T_1, T) = 0$$

Now
$$\frac{\partial C_T}{\partial T} = \frac{\partial C_{VT}}{\partial T} \times \left(\frac{1 - e^R}{1 - e^{RT}}\right) + C_{VT} \times \frac{\partial}{\partial T}\left(\frac{1 - e^R}{1 - e^{RT}}\right) \qquad (14)$$

In which

$$\frac{\partial C_{VT}}{\partial T} = \frac{-1}{nT^{\frac{n+1}{n}}} \times$$

$$\left[\frac{cd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)} - \frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right]\right] +$$

$$\frac{icd}{T^{1/n}}\left[\left(T_1 + \frac{RT_1^2}{2} - \frac{\theta T_1^3}{6} - \frac{R\theta T_1^4}{8}\right)T_1^{1/n} - \left(\frac{nT_1^{\frac{1+n}{n}}}{n+1} + \frac{nRT_1^{\frac{1+2n}{n}}}{2n+1}\right.\right.$$

$$\left. - \frac{R\theta nT_1^{\frac{1+4n}{n}}}{4n+1}\right)$$

$$\left. + \frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}} + \frac{RT_1^{\frac{1+4n}{n}}}{2} - \frac{\theta T_1^{\frac{1+5n}{n}}}{6} - \frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right)\right]$$

$$+ \frac{cd}{T^{1/n}}\left[\frac{(i_p - r)T_1^{1/n}(T^2 - M^2)}{2}\left\{1 - \frac{\theta}{4}\left(T_1^2 + M^2\right) + \frac{\theta T_1^2}{1+2n}\right\} - (i_p - r)n\right.$$

$$\left. -(i_p - r)n\left\{\frac{\left(T_1^{\frac{2n+1}{n}} - M^{\frac{2n+1}{n}}\right)}{2n+1} - \frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}} - M^{\frac{4n+1}{n}}\right)}{4n+1} + \frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}} - M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)}\right\}\right]$$

-

$$\frac{cd}{T^{1/n}}\left[(i_e - r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1} + (i_e + r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right.$$

$$\left. + \frac{c_b d}{T^{1/n}}(T - T_1)^{\frac{1}{n}}(1 + RT)\right] +$$

$$\frac{c_b d}{nT^{1/n}}(T - T_1)^{\frac{1-n}{n}}(1 + RT) +$$

$$\frac{c_b d}{T^{1/n}}(T - T_1)^{\frac{1}{n}} R = S$$

And $\frac{\partial}{\partial T}\left(\frac{1 - e^R}{1 - e^{RT}}\right) = \left(\frac{1 - e^R}{(1 - e^{RT})^2}\right)R$

Therefore

$$\frac{\partial C_T}{\partial T} = S \times \left(\frac{1 - e^R}{1 - e^{RT}}\right) + C_{VT} \times \frac{\partial}{\partial T}\left(\frac{1 - e^R}{1 - e^{RT}}\right) \qquad (15)$$

which is expressed in known quantities from equation 12, similarly

$$\frac{\partial C_T}{\partial T_1} = \frac{\partial C_{VT}}{\partial T_1} \times \left(\frac{1 - e^R}{1 - e^{RT}}\right) \qquad (16)$$

Solution of equation 15 and 16 will yield optimal T and T_1.

Optimal Solution

By using techniques of calculus it can be shown that equation 12 is convex in feasible domain of T and T_1. Therefore the optimum value of T and T_1 minimizing C_T can be obtained by simultaneously solving equations $\frac{\partial C_T}{\partial T_1} = 0$

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

and $\frac{\partial C_T}{\partial T} = 0$. The expression for the total cost involves higher order terms, it is not easy to evaluate the Hessians in closed form, to conclude about its positive definiteness directly, and thus it is not trivial to see whether the total cost function is convex.

Case II: T<M (i.e., after depletion payment)

The deterioration cost C_D^i, carrying cost, C_H^i and the backorder cost C_B^i per cycle are the same as in the equation 6, 7 and 10 respectively. The interest rate per cycle is equal to zero when T_1 <M because the supplier can pay in full at the end of permissible delay, M. The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during the time period (T ,M) after the inventory is exhausted at time T , it is given by

$$I_T^i = \int_0^T c_o I_e(t) t^{\frac{1}{n}-1} \frac{d}{nT^{1/n}} dt + (e^{i_e(M-T_1)} - 1)\int_0^{T_1} \frac{c_0 t dt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}} dt$$

$$= \frac{cd}{T^{1/n}}\left[(i_e - r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1} + (i_e + r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right] + (e^{i_e(M-T_1)} - 1)\frac{cd}{T^{1/n}}\left[\frac{T_1^{\frac{1+n}{n}}}{n+1} + \frac{RT_1^{\frac{1+2n}{n}}}{2n+1}\right] \quad (17)$$

Incorporating the modification of in equation 17 and P_T^i =0 into equation 11 because of the changes in assumption for case II, value in equation has changed from that in equation 12. The solution structure for the total annual cost remains the same as in equation 13. So a similar solution structure may be applied for the optimal solution of T_1 and T as it was done earlier from multiple cycles per year in equations 13-16.

IV. Mathematical Model and Analysis for Probabilistic Case

As in most of the practical situations demand is not deterministic, but it varies from cycle to cycle. Therefore, we have considered the probabilistic version of this model with demand equal to $\frac{xdt^{\frac{(1-n)}{n}}}{nT^{\frac{1}{n}}}$ where $(0 < x < \infty)$, thus demand considered here is probabilistic power demand. The differential equation for above demand pattern is

$$\frac{dI(t)}{dt} + \theta t I(t) = \frac{-xdt^{(1-n)/n}}{nT^{1/n}} \quad 0<t<T_1 \quad (18)$$

$$I(t).e^{\frac{\theta t^2}{2}} = \frac{-xd}{T^{1/n}}\left[t^{1/n} + \frac{\theta}{2}\frac{t^{1+2n/n}}{(2n+1)}\right] + C_2$$

Where at t = 0, I (t) = I_0. Putting this value in the above equation, we get

$I_0 = C_1$ This gives

$$I(t) = I_0 e^{\frac{-\theta t^2}{2}} - \frac{dxe^{\frac{-\theta t^2}{2}}}{T^{1/n}}\left[t^{1/n} + \frac{\theta}{2}\frac{t^{1+2n/n}}{(1+2n)}\right] \quad (19)$$

It is obvious that at t = T_1, I (T_1) = 0. So equation 19 yields

$$I_0 = \frac{xd}{T^{1/n}}\left[T_1^{1/n} + \frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right] = Q - b \quad (20)$$

Here b being the maximum backorder (Shortage) level permitted in each cycle.

Substituting the above value of I_0 in equation 18, we get

$$I(t) = \frac{xde^{\frac{-\theta t^2}{2}}}{T^{1/n}}\left[\left(T_1^{1/n} - t^{1/n}\right) + \frac{\theta}{2}\frac{\left(T_1^{1+2n/n} - t^{1+2n/n}\right)}{(1+2n)}\right]$$

$$0 < t < T_1 \quad (21)$$

Since shortages occur, we must have

$$I(T) < 0$$

$$\frac{xde^{\frac{-\theta T^2}{2}}}{T^{1/n}}\left[\left(T_1^{1/n} - T^{1/n}\right) + \frac{\theta}{2}\frac{\left(T_1^{1+2n/n} - T^{1+2n/n}\right)}{(1+2n)}\right] < 0 \quad (22)$$

And I (t) = 0 when, $T_1 \le t \le T$. The total demand during T_1 is $\int_0^{T_1} \frac{dxt^{\frac{1}{n}-1}}{nT^{1/n}} dt$. The amount of items which deteriorates during one cycle is given by

$$I_0 = \frac{dx}{T^{1/n}}\left[T_1^{1/n} + \frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right] = Q - b$$

$$D_T = I_o - \int_0^{T_1} \frac{dxt^{\frac{1}{n}-1}}{nT^{1/n}} dt = \frac{dx}{T^{1/n}}\left[\frac{\theta T_1^{\frac{2n+1}{n}}}{2(2n+1)}\right] \quad (23)$$

The Inflation Model

There are two distinct cases in this type of inventory system

I. Payment at or before the total depletion of inventory($M \le T_1 < T$)

II. After depletion payment($T_1 < M$)

Case I (i.e., payment at or before the total depletion of inventory)

(c) Since the ordering is made at time t=0, the inflation does not affect the ordering cost. Thus the ordering cost for items is fixed at A Dollars/order

(d) Since $I_0 = \frac{xd}{T^{1/n}}\left[T_1^{1/n} + \frac{\theta}{2}\frac{T_1^{1+2n/n}}{(1+2n)}\right]$, the value of this inventory at time t=0, is c I_0. The present value of the items sold is $\int_0^{T_1} \frac{c_0 xdt^{\frac{1}{n}-1}}{nT^{1/n}} dt$. Hence the cost of deterioration per cycle time T under inflation, C_D^i is given by

$$C_D^i = cI_0 - \int_0^{T_1} \frac{c_0 xdt^{\frac{1}{n}-1}}{nT^{1/n}} dt = \frac{cxd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)} - \frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right] \quad (24)$$

(d) The holding cost under inflation is given by

$$C_H^i = i\int_0^{T_1} c_0 I(t) dt$$

$$= \frac{icxd}{T^{1/n}}\left[\left(T_1 + \frac{RT_1^2}{2} - \frac{\theta T_1^3}{6} - \frac{R\theta T_1^4}{8}\right)T_1^{1/n} - \left(\frac{nT_1^{\frac{1+n}{n}}}{n+1} + \frac{nRT_1^{\frac{1+2n}{n}}}{2n+1} - \frac{\theta n T_1^{\frac{1+3n}{n}}}{3n+1} - \frac{R\theta n T_1^{\frac{1+4n}{n}}}{4n+1}\right)\right.$$

$$+ \frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}} + \frac{RT_1^{\frac{1+4n}{n}}}{2} - \frac{\theta T_1^{\frac{1+5n}{n}}}{6} - \frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right)$$

$$\left. - \frac{\theta}{2(1+2n)}\left(\frac{nT_1^{\frac{1+3n}{n}}}{3n+1} + \frac{nRT_1^{\frac{1+4n}{n}}}{4n+1} - \frac{\theta n T_1^{\frac{1+5n}{n}}}{5n+1} - \frac{R\theta n T_1^{\frac{1+6n}{n}}}{6n+1}\right)\right] \quad (25)$$

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The interest payable rate at time t is $e^{I_p t}-1$ dollars per dollar; so the present value (at t=0) of interest payable rate at time t is $I_p(t)=\left(e^{I_p t}-1\right)e^{-rt}$ rupees per rupees. Therefore the interest payable per cycle for the inventory not sold after the due date M is given by

$$P_T^i=\int_M^T cI_p(t)I(t)dt \qquad =\int_M^{T_1} cI_p(t)I(t)dt$$

I (t) = 0 for $T_1 \le t \le T$

$$=\frac{cxd}{T^{1/n}}\left[\frac{(i_p-r)T_1^{1/n}(T^2-M^2)}{2}\left\{1-\frac{\theta}{4}\left(T_1^2+M^2\right)+\frac{\theta T_1^2}{1+2n}\right\}-(i_p-r)n\right.$$

$$-(i_p-r)n\left\{\frac{\left(T_1^{\frac{2n+1}{n}}-M^{\frac{2n+1}{n}}\right)}{2n+1}-\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{4n+1}+\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)}\right\} \tag{26}$$

Where $i_p = I_p - r$

Similarly, the present value of the interest earned at time t, $I_e(t)$ is $\left(e^{I_e t}-1\right)e^{-rt}$. Considering inflated unit cost at time t as $c_t = ce^{Rt}$, the interest earned per cycle, I_T^i is the interest earned up to time T_1 and it is given by

$$I_T^i=\int_0^T c_o I_e(t)t\,d\,dt=\frac{cxd}{T^{1/n}}\left[(i_e-r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1}+(i_e+r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right] \tag{27}$$

The backorder cost per cycle under inflation, C_B^i is given by

$$C_B^i=\int_0^{T-T_1} c_b e^{R(T_1+t)}\frac{dxt^{\frac{1}{n}-1}}{nT^{1/n}}dt$$

$$=\frac{c_b xd}{T^{1/n}}(T-T_1)^{\frac{1}{n}}(1+RT) \tag{28}$$

Since the backorder starts at t=T_1

The Variable Cost Function

The total variable cost per cycle C_{VT}, is defined as

$$C_{VT}(T_1,T)=A+C_D^i+C_H^i+P_T^i-I_T^i+C_B^i \tag{29}$$

Substituting the values from equation 23-28 in equation 29, we have C_{VT} in terms of T_1 and T

$$C_{VT}(T_1,T)=A+\frac{cxd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)}-\frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right]+$$

$$\frac{icxd}{T^{1/n}}\left[\left(T_1+\frac{RT_1^2}{2}-\frac{\theta T_1^3}{6}-\frac{R\theta T_1^4}{8}\right)T_1^{1/n}-\left(\frac{nT_1^{\frac{1+n}{n}}}{n+1}+\frac{nRT_1^{\frac{1+2n}{n}}}{2n+1}-\frac{\theta nT_1^{\frac{1+3n}{n}}}{3n+1}-\frac{R\theta nT_1^{\frac{1+4n}{n}}}{4n+1}\right)\right.$$

$$+\frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}}+\frac{RT_1^{\frac{1+4n}{n}}}{2}-\frac{\theta T_1^{\frac{1+5n}{n}}}{6}-\frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right)$$

$$-\frac{\theta}{2(1+2n)}\left(\frac{nT_1^{\frac{1+3n}{n}}}{3n+1}+\frac{nRT_1^{\frac{1+4n}{n}}}{4n+1}-\frac{\theta nT_1^{\frac{1+5n}{n}}}{5n+1}-\frac{R\theta nT_1^{\frac{1+6n}{n}}}{6n+1}\right)$$

$$\frac{cxd}{T^{1/n}}\left[\frac{(i_p-r)T_1^{1/n}(T^2-M^2)}{2}\left\{1-\frac{\theta}{4}\left(T_1^2+M^2\right)+\frac{\theta T_1^2}{1+2n}\right\}-(i_p-r)n\right.$$

$$-(i_p-r)n\left\{\frac{\left(T_1^{\frac{2n+1}{n}}-M^{\frac{2n+1}{n}}\right)}{2n+1}-\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{4n+1}+\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)}\right\}-$$

$$\frac{cxd}{T^{1/n}}\left[(i_e-r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1}+(i_e+r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right]$$

$$+\frac{c_b xd}{T^{1/n}}(T-T_1)^{\frac{1}{n}}(1+RT) \tag{30}$$

Multiple Cycles per Year

Effect of inflation and time value of money is considered for each cycle of replenishment, so we need to consider the effect over the time horizon NT. Let there be N complete cycle during a year. Hence, NT=1. The total variable cost during total time is given by

$$C_T=C_{VT}\times\left[1+e^{2RT}+e^{3RT}+\ldots\ldots\ldots+e^{(N-1)RT}\right]$$

$$=C_{VT}\times\left[\frac{1-e^{NRT}}{1-e^{RT}}\right]=C_{VT}\times\left[\frac{1-e^{R}}{1-e^{RT}}\right] \tag{31}$$

The value of T and T_1 which minimize C_T may be obtained by simultaneously solving

$\frac{\partial C_T}{\partial T}(T_1,T)=0$ and $\frac{\partial C_T}{\partial T_1}(T_1,T)=0$,Now

$$\frac{\partial C_T}{\partial T}=\frac{\partial C_{VT}}{\partial T}\times\left(\frac{1-e^{R}}{1-e^{RT}}\right)+C_{VT}\times\frac{\partial}{\partial T}\left(\frac{1-e^{R}}{1-e^{RT}}\right) \tag{32}$$

In which

$$\frac{\partial C_{VT}}{\partial T}=\frac{-1}{nT^{\frac{n+1}{n}}}\times\left[\frac{cxd}{T^{1/n}}\left[\frac{\theta T_1^{\frac{1+2n}{n}}}{2(2n+1)}-\frac{RT_1^{\frac{1+n}{n}}}{(n+1)}\right]+\right.$$

$$\frac{icxd}{T^{1/n}}\left[\left(T_1+\frac{RT_1^2}{2}-\frac{\theta T_1^3}{6}-\frac{R\theta T_1^4}{8}\right)T_1^{1/n}-\left(\frac{nT_1^{\frac{1+n}{n}}}{n+1}+\frac{nRT_1^{\frac{1+2n}{n}}}{2n+1}-\frac{\theta nT_1^{\frac{1+3n}{n}}}{3n+1}-\frac{R\theta nT_1^{\frac{1+4n}{n}}}{4n+1}\right)\right.$$

$$+\frac{\theta}{2(1+2n)}\left(T_1^{\frac{1+3n}{n}}+\frac{RT_1^{\frac{1+4n}{n}}}{2}-\frac{\theta T_1^{\frac{1+5n}{n}}}{6}-\frac{R\theta T_1^{\frac{1+6n}{n}}}{8}\right)$$

$$+\frac{cxd}{T^{1/n}}\left[\frac{(i_p-r)T_1^{1/n}(T^2-M^2)}{2}\left\{1-\frac{\theta}{4}\left(T_1^2+M^2\right)+\frac{\theta T_1^2}{1+2n}\right\}-(i_p-r)n\right.$$

$$-(i_p-r)n\left\{\frac{\left(T_1^{\frac{2n+1}{n}}-M^{\frac{2n+1}{n}}\right)}{2n+1}-\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{4n+1}+\frac{\theta}{2}\frac{\left(T_1^{\frac{4n+1}{n}}-M^{\frac{4n+1}{n}}\right)}{(2n+1)(4n+1)}\right\}$$

$$-\frac{cxd}{T^{1/n}}\left[(i_e-r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1}+(i_e+r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1}\right]$$

$$\left.+\frac{c_b xd}{T^{1/n}}(T-T_1)^{\frac{1}{n}}(1+RT)\right]$$

$$+\frac{c_b xd}{nT^{1/n}}(T-T_1)^{\frac{1-n}{n}}(1+RT)\frac{c_b xd}{T^{1/n}}(T-T_1)^{\frac{1}{n}}R=S_2$$

And $\frac{\partial}{\partial T}\left(\frac{1-e^{R}}{1-e^{RT}}\right)=\left(\frac{1-e^{R}}{(1-e^{RT})^2}\right)R$

Therefore,

$$\frac{\partial C_T}{\partial T}=S_2\times\left(\frac{1-e^{R}}{1-e^{RT}}\right)+C_{VT}\times\frac{\partial}{\partial T}\left(\frac{1-e^{R}}{1-e^{RT}}\right) \tag{33}$$

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

which is expressed in known quantities from equation 30, similarly

$$\frac{\partial C_T}{\partial T_1} = \frac{\partial C_{VT}}{\partial T_1} \times \left(\frac{1 - e^R}{1 - e^{RT}} \right) \quad (34)$$

Solution of equation 33 and 34 will yield optimal T and T_1.

Optimal Solution

By direct search approach it can be shown that equation 31 is convex in feasible domain of T and T_1. Therefore the optimum value of T and T_1 minimizing C_T can be obtained by simultaneously solving equations $\frac{\partial C_T}{\partial T_1} = 0$ and $\frac{\partial C_T}{\partial T} = 0$. The expression for the total cost involves higher order terms, it is not easy to evaluate the Hessians in closed form, to conclude about its positive definiteness directly, and thus it is not trivial to see whether the total cost function is convex.

Case II T<M (i.e., after depletion payment)

The deterioration cost C_D^i, carrying cost, C_H^i and the backorder cost C_B^i per cycle are the same as in the equation 24, 25 and 28 respectively. The interest rate per cycle is equal to zero when T_1 <M because the supplier can pay in full at the end of permissible delay, M. The interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during the time period (T ,M) after the inventory is exhausted at time T , it is given by

$$I_T^i = \int_0^T c_o I_e(t)\,tdxdt + (e^{i_e(M-T_1)} - 1)\int_0^{T_1} \frac{c_0\, tdxt^{\frac{1-n}{n}}}{nT^{\frac{1}{n}}}dt$$

$$= \frac{cd}{T^{1/n}} x \left[(i_e - r)\frac{T_1^{\frac{1+2n}{n}}}{2n+1} + (i_e + r)\frac{RT_1^{\frac{1+3n}{n}}}{3n+1} \right]$$

$$+ (e^{i_e(M-T_1)} - 1)\frac{cxd}{T^{1/n}}\left[\frac{T_1^{\frac{1+n}{n}}}{n+1} + \frac{RT_1^{\frac{1+2n}{n}}}{2n+1} \right] \quad (35)$$

Incorporating the modification of in equation 35 and P_T^i =0 into equation 29 because of the changes in assumption for case II, value in equation has changed from that in equation 30. The solution structure for the total annual cost remains the same as in equation 31. So a similar solution structure may be applied for the optimal solution of T_1 and T as it was done earlier from multiple cycles per year in equations 31-34.

V. CONCLUSION

This study is devoted to products having power demand pattern with variable rate of deterioration under the effect of inflation. As it is not possible to forecast exact demand in advance, therefore we have considered both probabilistic and deterministic versions of these models. In probabilistic model the demand pattern remains same but it depends upon the value of x which may vary from 0 to ∞. Items considered in this study are perishable items (especially fruits and vegetables) with variable rate of deterioration. This study is unique in itself, as it is the first time that inflation and time value of money has been considered with power demand pattern under permissible delay in payments.

In this study, we have considered two cases with respect to depletion time T_1 which is either greater than or less than equal to permissible delay. Thus, these models will help inventory managers in deciding their stock of inventory for perishable items with time dependent demand rates. These models although prepared for retailers but can also be used by both suppliers and retailers. In these models effect of time value of money and inflation are also considered which are important part of today's business environment. As most of the items which, we considered here are retailer's items, therefore shortages are allowed, because they cannot store huge stock of these items.

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ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)