

# Model of Integrated Production-Inventory-Distribution System: The Case of Billet Steel Manufacturing

Parwadi Moengin and Rina Fitriana, *Member, IAENG*

**Abstract**—A significant phase of billet steel manufacturing is the cooling process. This phase may be completed in the finished goods warehouse and it must meet both production optimization and customer needs. The fluctuation of customer demand and production process in this phase may cause some problems of production schedule and inventory of billet steel. The decreasing of customer demand can increase the inventory level in finished goods warehouse. Therefore, the company need to consider a production planning that gives an optimum decision of billet steel production schedule. In this paper, a mathematical model of integrated production-inventory-distribution system of billet steel will be proposed. The model is used to find the optimal production schedule of billet steel, based on the relevant parameters of the productive system, such as: set-up time, processing times, and demand profile. The study is completed by a case study to investigate the proposed model.

**Index Terms**—production-inventory-distribution system, scheduling, linear programming, mathematical model.

## I. INTRODUCTION

This paper discusses the optimal method of production scheduling in billet steel products ordered by consumers considering the limitations of space in the warehouse. Fluctuations in consumer demand can be considered as one of the causes of some problems in the production and supply company. Increased demand led to problems concerning storage capacity cooling billet steel products and raw material supply of iron ore. Decline in demand cause overproduction and excessive accumulation of finished goods. This poses particular problems in the cooling warehouse capacity of billet steel products. Observing this, companies need to consider the production planning so that activities are carried out more optimal production to meet market demand. Production planning concerns the tactical planning provides optimum decisions based on the availability of time and the capacity of the company's inventory to overcome problems related to the production process.

Utilization of mathematical model for the optimization of production, especially the production of billet steels has been done by many researchers, including Chen and Wang (1997) which uses linear programming models for production planning and distribution of steel, Kapusinski and Tayur (1998) model the production system limited to periodic demand and Kalagnanam et al. (2000) addressed the issue of excess inventory in the process industry.

Manuscript is submitted on March 8, 2015. This research was supported and funded by the Ministry of Education and Culture, Republic of Indonesia.. Parwadi Moengin and Rina Fitriana are with the Department of Industrial Engineering, Faculty of Industrial Technology, Trisakti University, Jakarta 11440, Indonesia. (email: parwadi@trisakti.ac.id).

Several other authors such as Lally et al. (1997) have made a model of sequencing a continuous casting operation in order to minimize costs, Mohanty and Singh (1998) using a hierarchical approach to planning system steel manufacturing, Tang et al. (2000) using a mathematical programming model for scheduling the production of continuous casting of steel and Tang et al. (2001, 2002) have discussed the problem of production planning and scheduling using Lagrange relaxation. Zanoni et. al. latter. (2005) discuss the optimization of the production system of billet steel products.

In general, the billet steel production process starting from (1) the process of melting and cooking through the stages: filling and blending sponge iron, iron and hot scrap bracket iron in the bucket; smelting in the Electric Arc Furnace; Oxidation processes Refining & Electric Arc Furnace; (2) Ladle Refining Furnace process on, and ends with (3) a process in Continuous Casting Machine. This paper is structured as follows. Writing begins from the introduction that discusses the optimization of production associated with distribution, scheduling and inventory that has been done by some previous authors; followed by the proposal of a model of integer linear programming for integrated production-inventory-distribution systems of billet steel products. A case study in PT. XYZ is used to apply the proposed model.

## II. MODEL FORMULATION FOR INTEGRATED PRODUCTION-INVENTORY-DISTRIBUTION SYSTEM

The model is formulated to focus on the optimization of production planning, i.e. the amount of production in the planning horizon. The mathematical model is focused on sustainable production, where finished goods warehouse capacity becomes an important part in the production cycle. To solve this problem, an integer linear programming models is introduced by taking into account the capacity of finished goods warehouse in continuous production planning.

In continuous production, initially satisfied the market demand of finished goods inventory in the warehouse, then just do the next planning to meet the rest of the reservation. In this case the integer linear programming models are used, has a goal to maximize profits at the end of the sale of products incorporating the costs saved in the finished goods warehouse, and also pay attention to the cost penalty as the costs incurred because the manufacturer cannot meet the market reservations on time.

Basically the integer linear programming model of this paper, consider three kinds of costs, namely:

- Saving costs, these costs directly affect the final profit of the company, according to the duration of storage of finished products in the finished goods warehouse.
- Production costs (which is one of its components is a set up cost)
- Penalty cost, as 'fine' company being unable to meet market reservations on time.

Model of integrated production-inventory-distribution systems is formulated with respect to some constraints, i.e. the production time, the quantity of products in the warehouse, production planning period, the suitability of the products sold by the products ordered in the planning, the quantity of product sales, and warehouse capacity. Decision variables of the model is the number of products to be produced, the number of products available in the warehouse, the number of products are delivered to consumers each period, as well as the decision whether or not to produce billet steel in the planned time period.

The following are the parameters of the mathematical model of integer linear programming for continuous production planning:

- $n$  : the number of product type  
 $m$  : planning time period (in months/weeks/ days)  
 $i$  : indeks of product ( $i = 1, \dots, n$ )  
 $j$  : indeks of time ( $j = 1, \dots, m$ )  
 $h$  : production planning in the future (12 months)  
 $s_i$  : surface area product type  $i$   
 $NBC_i$  : maximum production capacity of product type  $i$   
 $Pt$  : required production time  
 $H_d$  : available production time (= 28 days / month for 12 months / year)  
 $I_c$  : holding cost per print products per day  
 $P_r$  : average profit per mold products  
 $St$  : average *set up* time at machine EAF  
 $O_{ij}$  : demand quantity of product  $i$  in period  $j$   
 $L_{inv}$  : length of warehouse  
 $I_{oi}$  : initial inventory in the warehouse of product  $i$   
 $M$  : large numbers (been greater than the amount of product that can be produced per period)  
 $aux_i$  : auxiliary variable that represents how much space is required to store the products  $i$   
 $C_s$  : penalty cost per unit for late delivery in quarter  $s$  ( $s = 1, 2, 3, 4$ )  
 $P_{ij}$  : decision variables, the number of product  $i$  produced in the period  $j$   
 $I_{ij}$  : decision variables, the number of products available in the cooling warehouse during the period  $j$   
 $S_{ij}$  : decision variables, the number of products  $i$  are delivered to customers in the period  $j$   
 $y_{ij}$  : binary decision variable (set up machine EAF to produce billet tiper  $i$  in period  $j$ )

$$y_{ij} = \begin{cases} 1, & \text{if produce product } i \text{ at period } j \\ 0, & \text{otherwise} \end{cases}$$

The following is the formulation of the model for production-inventory-distribution system:  
Maximize:

$$Z = \left[ \begin{aligned} & \sum_{i=1}^n \sum_{j=1}^m (P_r \cdot S_{ij} - I_{ij} \cdot I_c \cdot H_d) \\ & - C_1 \cdot \sum_{i=1}^n \sum_{j=1}^{\frac{h}{4}} (O_{ij} - S_{ij}) \\ & - C_2 \cdot \sum_{i=1}^n \sum_{j=\frac{h}{4}+1}^{\frac{h}{2}} (O_{ij} - S_{ij}) \\ & - C_3 \cdot \sum_{i=1}^n \sum_{j=\frac{h}{2}+1}^{\frac{3h}{4}} (O_{ij} - S_{ij}) \\ & - C_4 \cdot \sum_{i=1}^n \sum_{j=\frac{3h}{4}+1}^h (O_{ij} - S_{ij}) \end{aligned} \right]$$

subject to:

$$\sum_{i=1}^n [Pt \cdot P_{ij} + St \cdot y_{ij}] \leq Hd, \forall j = 1, \dots, m \quad (1)$$

$$I_{i1} = I_{oi} + P_{i1} - S_{i1}, \forall i = 1, \dots, n \quad (2)$$

$$I_{ij} = I_{i,j-1} + P_{ij} - S_{ij}, \forall i = 1, \dots, n, j = 2, \dots, m \quad (3)$$

$$\sum_{j=1}^h P_{ij} = \sum_{j=1}^h (O_{ij} - I_{oi}), \forall i = 1, \dots, n \quad (4)$$

$$\sum_{j=1}^h S_{ij} = \sum_{j=1}^h O_{ij}, \forall i = 1, \dots, n \quad (5)$$

$$\sum_{d=1}^{j-1} S_{id} \leq \sum_{d=1}^{j-1} O_{id}, \forall i = 1, \dots, n, j = 2, \dots, m \quad (6)$$

$$P_{ij} \leq My_{ij}, \forall i = 1, \dots, n, j = 1, \dots, m \quad (7)$$

$$\sum_{i=1}^n I_{ij} \cdot s_i \cdot NBC_i \cdot aux_i \leq L_{inv}, \forall j = 1, \dots, m \quad (8)$$

$$P_{ij}, I_{ij}, S_{ij} \geq 0 \text{ integer}, y_{ij} \in \{0, 1\} \quad (9)$$

This model involves one goal and nine constraints. The objective function is to maximize the profit from the selling of billet steel by taking into account the inventory cost, production cost and penalty cost. In the models, the average profit per mold product ( $P_r$ ) represents the difference between the selling price per unit of the product with the production cost. In the objective function is assumed also that the number of delivered product shall not exceed the amount of the requested product, so  $O_{ij} - S_{ij} \geq 0$ . The nine constraints considered in the model are (1) describe the available production time of each month (the sum of the set-up time and the time of production must be less than one

month), (2) and (3) the quantity of products in the warehouse based on the type and moon, (4) the amount of production in the planning period (the difference between the amount of demand for products in the same period the number of initial inventory of products in the warehouse), (5) demonstrate the suitability of the products sold by the products ordered in the planning, (6) guarantee the quantity of products sold up to a period of months-d ( $d = 1, \dots, H$ ) less than or equal to the quantity to be delivered, (7) ensure that the model is linear binary variable  $y_{ij}$  allocated, as well as barrier (8) the capacity of the warehouse, where the products are stored for refrigerated warehouse cannot exceed available capacity. Constraint (9) is limiting for the decision variables.

### III. CASE STUDY

Here is presented a case study on PT. XYZ to implement the proposed model. PT. XYZ is an integrated steel industry in Indonesia with a production capacity of 2.5 million tons of crude steel per year. PT. XYZ produces various kinds of products according to customer demand. The resulting product is a sponge iron, steel slab, billet steel, hot rolled coils, cold rolled steel, and wire rod. In this paper focused on the production of billet steels which consists of three types, namely billet steel 110, billet steels 120 and billet steel 130. This paper presents an attempt to optimize the production planning process cooling products ranging from billet steel to billet steel product deliveries to customers by taking into account the limited capacity of finished goods warehouse. The data needed is a billet of steel product sales data for the two previous years are used to forecast the demand for billet steel products in the coming year, the data amount of the initial inventory of finished goods in the warehouse, inventory cost data, the cost penalty, the average profit per year, wide warehouse, set up time and production capacity.

Here is a table summarizing the results of forecasting demand by using three types of forecasting methods for billet steel products:

TABLE 1  
THE RESULTS OF ALL THREE TYPES OF DEMAND FOR  
BILLET STEEL PRODUCTS

Month	Type of product (sheet)		
	Steel billet 110	Steel billet 120	Steel billet 130
January	1.837	2.448	15.597
February	1.763	2.350	14.969
March	1.696	2.260	14.401
April	1.641	2.187	13.931
May	1.601	2.133	13.591
June	1.579	2.104	13.404
July	1.577	2.101	13.383
August	1.594	2.124	13.529
September	1.630	2.171	13.833
October	1.681	2.240	14.274
November	1.746	2.326	14.821
December	1.818	2.423	15.438
Total	20.163	26.867	171.171

The objective of case study conducted in PT. XYZ is maximize profits from the sale of billet steels. The model is formulated involving 144 binary decision variables and 159 constraints. Therefore, the completion of the model is done with the help of software Microsoft Excel Solver. Initial inventory for all three types of billet steels each is 505 sheets, 457 sheets and 139 sheets, respectively. Table 2 is

the result of the calculation of the optimal production quantities.

TABLE 2  
OPTIMAL PRODUCTION OF BILLET STEEL

Month	Type of product (sheet)		
	Steel billet 110	Steel billet 120	Steel billet 130
January	0	1.991	15.458
February	3.379	0	14.969
March	0	4.739	1.124
April	3.053	0	16.627
May	1.885	4.320	12.995
June	0	0	19.680
July	0	4.076	15.604
August	3.180	124	16.136
September	3.200	4.300	11.940
October	1.475	2.111	15.854
November	1.907	2.326	15.207
December	1.579	2.423	15.438
Total	19.658	26.410	171.032

From the data of the production plan in next year, we compare the results obtained by calculating the optimal production of the model. The production plan of the next year in PT. XYZ is as follows:

TABLE 3  
PRODUCTION PLAN OF BILLET STEEL

Month	Type of product (sheet)		
	Steel billet 110	Steel billet 120	Steel billet 130
January	677	902	5.744
February	2.255	3.005	19.146
March	1.236	1.627	10.493
April	3.340	4.451	28.366
May	1.635	2.179	13.887
June	3.147	4.193	26.724
July	2.308	3.076	19.600
August	2.595	3.458	22.035
September	1.341	1.787	11.389
October	1.514	2.017	12.853
November	2.554	3.404	21.692
December	1.994	2.657	16.929
Total	24.596	32.776	208.858

The comparison between planned production is illustrated and the optimal steel production is for all three types of steel. Figure 1-2 show that the production plans of billet steel PT. XYZ is generally greater than the results of the calculation of the mathematical integer programming model production. The possibility of excess production would lead to a buildup of inventories of finished goods in the warehouse cooling. This will cause expenses, which increased storage costs, because of the possibility that the number of products shipped to consumers with unbalanced output, in which the amount of production is greater than the market demand.

The optimal inventory of each type of billet steel products is presented in Table 4. The results of these calculations, obtained the following results.

Table 5 describes the number of billet steel products for any type that is sent to the consumer. In January there were 505 pieces of billet delivery in the 110 even though there is no production, because there is still a beginning inventory that exceeds consumer demand.

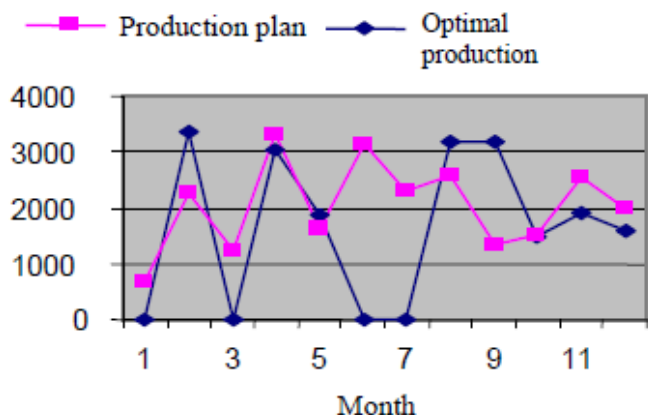


Figure 1 Production plan and optimal production of billet steel 110

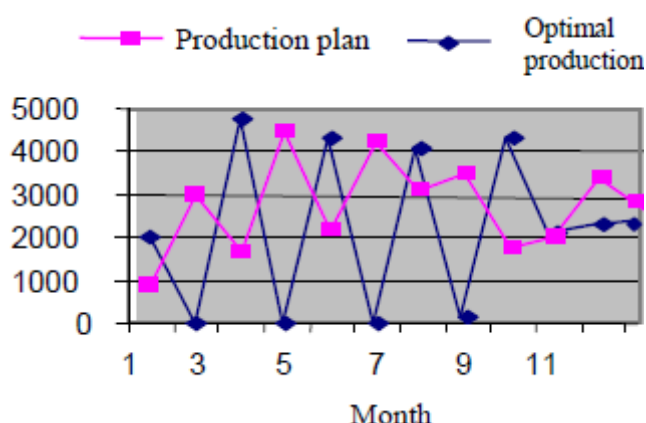


Figure 2 Production plan and optimal production of billet steel 120

TABLE 4.  
 THE OPTIMAL INVENTORY OF BILLET STEEL IN WAREHOUSE

Month	Type of product (sheet)		
	Steel billet 110	Steel billet 120	Steel billet 130
January	0	0	0
February	284	0	0
March	284	129	22
April	0	0	0
May	284	129	0
June	284	129	22
July	0	0	0
August	0	0	0
September	284	129	22
October	78	0	0
November	237	0	0
December	0	0	0

TABLE 5  
 THE NUMBER OF PRODUCT TO BE DELIVERED

Month	Type of product		
	Steel billet 110	Steel billet 120	Steel billet 130
January	505	2.448	15.597
February	3.095	0	14.969
March	0	4.610	1.102
April	3.337	129	16.649
May	1.601	4.191	12.995
June	0	0	19.658
July	284	4.205	15.626
August	3.180	124	16.136
September	2.916	4.171	11.918
October	1.681	2.240	15.876
November	1.748	2.326	15.207
December	1.816	2.423	15.438
Total	20.163	26.867	171.171

The total number of billet steel products are shipped within one year for each species (Table 5) exactly the same as the number of products demanded by consumers (Table 1); this case shows that the model has been valid. From Table 5 also shows that the delivery of billet steel 110 and 120, respectively do not have to be done every month,

From the calculation, it is known that in January, March, June, and July do not need to be set-up machine EAF to produce billet steel 110, which is in line with the production of billet steel 110, which in the fourth month, do not do production for billet steel 110. In February, April, and June did not do the set-up machine EAF to produce billet steel 120, for the three months was not done for the production of billet steel 120. Based on the results over the planning horizon of twelve months, the obtained gains production of billet steel for one year is \$ 113.5 million.

#### IV. CONCLUSION

In this paper we propose a mathematical model for production planning in the process of making billet steels. Starting from the case of billet steel industry, the proposed model considers the amount of billet steels available in the previous month, consumer demand in the current month and billet cooling area in the warehouse as an integrated part of the system of billet steel production. Microsoft Excel Solver software developed to optimize the production schedule.

For that reason in producing billet steel production should be done at every period, so that the company can optimize the use of warehouse / finished goods warehouse for the cooling process to the delivery of billet steel products to consumers. Thus, companies can minimize the expenditure in terms of cost savings and cost penalties. Future studies directed to involve patterns of raw material supply in a model system of production-inventory-distribution.

#### REFERENCES

- [1] Chen, M., and Wang, W., 1997. "A linear programming model for integrated steel production and distribution planning." *International Journal of Operations & Production Management*, Vol. 17, No.6, p.592-610.
- [2] Kalagnanam, J.R., Dawande, M.W., Trumbo, M., and Lee, H.S., 2000. "The surplus inventory matching problem in the process industry." *Operations Research*, Vol. 48, No. 4, p.505-516.
- [3] Kapusinski, R., and Tayur, S., 1998. "A capacitated production inventory model with periodic demand." *Operations Research*, Vol. 46, No. 6, p.899-911.
- [4] Lally, B., Biegler, L., and Henein, H., 1997. "A model for sequencing a continuous casting operation to minimize costs." *Iron & Steelmaker*, Vol 10, p.63-70.
- [5] Mohanty, R.P., and Singh, R., 1998. A hierarchical production planning approach for a steel manufacturing system. *International Journal of Operation s & Production Management*, Vol. 12, No. 5, p.69-78.
- [6] Tang, L., Liu, J., Rong, A., and Yang, Z., 2000. "A mathematical programming model for scheduling steelmaking continuous casting production." *European Journal of Operational Research*, Vol. 120, No. 2, p.423-435.
- [7] Tang, L., Liu, J., Rong, A., and Yang, Z., 2001. "A review of planning and scheduling systems and

- methods for integrated steel production.” *European Journal of Operational Research*, Vol. 133, No. 10, p.1–20.
- [8] Tang, L., Luh, P.B., Liu, J., and Fang, L., 2002. “Steel-making process scheduling using Lagrangian relaxation.” *International Journal of Production Research*, Vol. 40, No. 1, p.55–70.
- [9] Widjaja, Audrey Margareta (2006). *Aplikasi model integer programming dalam sistem produksi-persediaan-distribusi terpadu produk baja billet pada PT. Krakatau Steel*. Tugas Akhir. Jurusan Teknik Industri Universitas Trisakti Jakarta
- [10] Zanoni, Simone and Lucio Zavanella (2005). “Model and Analysis of Integrated Production-Inventory System: The Case of Steel Production.” *International Journal of Production Economics*, Vol. 93-94, p.197 – 205.