On the Exponentiated Generalized Inverse Exponential Distribution

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***Abstract*— This research explored the Exponentiated Generalized Inverse Exponential (EGIE) distribution to include more statistical properties and in particular, applications to real life data as compared with some other generalized models.**

***Index Terms*— Data, Generalization, Inverse Exponential, Statistical Properties**

I INTRODUCTION

The Inverse Exponential (IE) distribution is a life time model which is capable of modeling real life phenomena with bathtub failure rates. The model is a modification of the Exponential distribution as it accounts for a short-coming of the Exponential distribution (a model with constant failure rate). The application of the model can be found in biology, engineering and medicine. Details about the IE distribution can be found in [1], [2], [3] and [4].

Various generalized models have been proposed in recent years and their flexibilities over their baseline distributions when applied to real life data have been established. The Beta-Gompertz distribution, Weibull-Exponential distribution, Transmuted Exponential distribution, Beta-Nakagami distribution and Kumaraswamy-Dagum distribution are notable examples in the literature. see [5], [6], [7], [8] and [9] respectively. Details about various classes of generalized distributions can be found in [10] and [11].

Attempts to increase the flexibility of the IE distribution birthed the Generalized Inverted Exponential (GIE) distribution; [12], Exponentiated Generalized Inverted Exponential (EGIE) distribution; [13], Kumaraswamy Inverse Exponential (KIE) distribution; [14] and Transmuted Inverse Exponential (TIE) distribution; [15]. But, of interest to us in this research is the EGIE distribution. The reason being that, [10] demonstrated that the Exponentiated Generalized Gumbel (EGGu) distribution is more flexible than the Beta Gumbel and Kumaraswamy Gumbel distributions based on application to real life data.

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This article is therefore aimed at demonstrating the usefulness of the EGIE distribution using three real life data in order to assess its flexibility over its sub-models and some other generalized models.

II THE EXPONENTIATED GENERALIZED INVERSE EXPONENTIAL (EGIE) DISTRIBUTION

The EGIE distribution was derived using the Exponentiated Generalized family of distributions due to [16]. The following models were also derived using the same concept; The Exponentiated Generalized Frechet distribution, Exponentiated Generalized Normal distribution, Exponentiated Generalized Gamma distribution, Exponentiated Generalized Gumbel distribution; see [16], Exponentiated Generalized Inverse Weibull distribution; [17] and Exponentiated Generalized Weibull distribution; [18].

The cdf and the pdf of the EGIE distribution with parameters α, β and θ are given by;

$$F(x) = \left[1 - \left\{1 - \exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta} \quad (1)$$

and

$$f(x) = \alpha\beta\theta x^{-2} \exp\left(-\frac{\theta}{x}\right)\left\{1 - \exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha-1} \times \left[1 - \left\{1 - \exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta-1} \quad (2)$$

respectively

For $x > 0, \alpha > 0, \beta > 0, \theta > 0$

where; α and β are shape parameters whose role are to vary tail weights

θ is a scale parameter

Further Properties of the EGIE distribution

Some properties of the EGIE distribution are available in [13] but here, some further basic properties are provided in some details.

Asymptotic Behavior

The behavior of the cdf of the EGIE distribution is being investigated as $x \to \infty$.

Lemma 1: It can be shown that for EGIE distribution, $\lim_{x\to\infty} F(x) = 1$

Proof:

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty}\left\{\left[1-\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta}\right\}$$

$$=\left\{\left[1-\left\{1-\exp\left(-\frac{\theta}{\infty}\right)\right\}^{\alpha}\right]^{\beta}\right\}$$

Therefore;

$$\lim_{x\to\infty} F(x) = \left\{\left[1-(0)^{\alpha}\right]^{\beta}\right\} = 1$$

Odds Function

Odds function is derived by:

$$O(x) = \frac{F(x)}{S(x)}$$

Therefore, the odds function for the EGIE distribution is given by:

$$O(x) = \frac{\left[1-\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta}}{1-\left[1-\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta}} \quad (3)$$

Reversed Hazard Function

This can be obtained from:

$$r(x) = \frac{f(x)}{F(x)}$$

Therefore, the reversed hazard function for the EGIE distribution is given by:

$$r(x) = \frac{\alpha\beta\theta x^{-2}\exp\left(-\frac{\theta}{x}\right)\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha-1}}{\left[1-\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]} \quad (4)$$

III APPLICATION

In this section, the EGIE distribution is applied to three real life datasets, the aim is to assess its flexibility over its baseline distribution and some other generalized models. The models compared in this research are; Kumaraswamy Inverse Exponential (KIE) distribution, Generalized Inverse Exponential (GIE) distribution and Inverse Exponential (IE) distribution. The model with the lowest Alkaike Information Criteria (AIC) or the largest Log-likelihood value is regarded as the best. The pdf of the models under study are given in Table 1;

DATA I: The first data represents the real life times (in minutes) of patients receiving an analgesic. The data was given by [19]. The summary of the data is given in Table 2 and the performances of the selected models are given in Table 3.

DATA II: The second data set represents the life of fatigue fracture of Kevlar 373/epoxy subjected to constant pressure at 90% stress level until all had failed. The data has been previously studied by [20], [21] and [22]. The summary of the data is given in Table 4 and the performances of the selected models are given in Table 5.

DATA III: The third data represents the number of million revolutions before failure for each of 23 deep groove ball bearings in the life tests. The data was given by [23]. The summary of the data is given in Table 6 and the performances of the selected models are given in Table 7.

Table 1: The pdfs of competing models

Models	**Pdf**
EGIE distribution	$f(x) = \alpha\beta\theta x^{-2}\exp\left(-\frac{\theta}{x}\right)\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha-1}\left[1-\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta-1}$
KIE distribution	$f(x) = \alpha\beta\theta x^{-2}\exp\left(-\frac{\theta}{x}\right)\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha-1}\left[1-\left\{\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha}\right]^{\beta-1}$
GIE distribution	$f(x) = \alpha\theta x^{-2}\exp\left(-\frac{\theta}{x}\right)\left\{1-\exp\left(-\frac{\theta}{x}\right)\right\}^{\alpha-1}$
IE distribution	$f(x) = \frac{\theta}{x^2}\exp\left(-\frac{\theta}{x}\right)$

Table 2: Summary of data on relief times of patients

n	mean	variance	skewness	kurtosis
20	1.90	0.4958	1.5924	5.9241

Table 3: Model Performance with respect to DATA I

Models	Estimates	LL	AIC
EGIE	$\alpha = 4.6762,$ $\beta = 484.5728,$ $\theta = 0.4869$	-15.4	36.8
KIE	$\alpha = 0.0457,$ $\beta = 20.7662,$ $\theta = 134.9988$	-17.1	40.2
GIE	$\alpha = 20.766, \theta = 6.171$	-17.1	38.2
IE	$\theta = 1.7247$	-32.7	67.3

Table 4: Summary of data on life of fatigue fracture of Kevlar 373/epoxy

n	mean	variance	skewness	kurtosis
76	1.9590	2.4774	1.9406	8.1608

Table 5: Model Performance with respect to DATA II

Models	Estimates	Log likelihood	AIC
EGIE	$\alpha = 7.595 \times 10^{-1},$ $\beta = 1.446 \times 10^{3},$ $\theta = 5.649 \times 10^{-5}$	-153.6	313.2
KIE	$\alpha = 24.1684,$ $\beta = 0.7904,$ $\theta = 0.0216$	-161.9	329.9
GIE	$\alpha = 0.79036,$ $\theta = 0.52254$	-161.9	327.9
IE	$\theta = 0.62487$	-163.1	328.2

Table 6: Summary of data on ball bearings

n	mean	variance	skewness	kurtosis
23	72.23	1404.783	0.8812	3.4889

Table 7: Model Performance with respect to DATA III

Models	Estimates	LL	AIC
EGIE	$\alpha = 2.0621,$ $\beta = 27.2256,$ $\theta = 10.7126$	-115.3	236.6
KIE	$\alpha = 27.069,$ $\beta = 5.314,$ $\theta = 4.806$	-113.5	233.1
GIE	$\alpha = 5.314,$ $\theta = 130.096$	-113.5	231.1
IE	$\theta = 55.074$	-121.7	245.5

IV CONCLULSION

The study on EGIE distribution has been successfully extended to include real life applications. The EGIE distribution is capable of modeling data sets that are positively skewed and that have inverted bathtub failure rates. The model was found to be better than competing models (Kumaraswamy Inverse Exponential distribution, Generalized Inverse Exponential distribution and Inverse Exponential distribution) except for data sets where the variance is far larger than the mean. The model can be successfully be used to model lifetime data sets and real life phenomena with inverted bathtub failure rates.

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