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Application for Logical Eduction of Discrete Mathematics — Hierarchical Approach —

Kazuyoshi MORI Kensuke SHINANO Takahiro TSUCHIYA

***Abstract*—In this paper, we will report the implementation of an educational system based on the hierarchical approach. The hierarchical approach is a logical approach, which will proceed "what we need to show" and "Its solution" successively. The set equation is employed for the demonstration. The system is implemented on iOS.**

***Index Terms*—Logical Education, Hierarchical Approach, Discrete Mathematics, Set Equation,**

I. Introduction

When we study mathematics of the school education in elementary school, junior high school, and high school in Japan, logicality is desired repeatedly[1], [2], [3]. We consider that logicality is one of the most important studying mathematics of the school education.

First of all, we introduce existing various applications for education.

E-Text(electric text)[4], [5] is general term for any document that is read in digital form, and especially a document that is mainly an educational text. A computer based book of art with minimal text, or a set of photographs or scans of pages, would not usually be called an "e-text." The term is usually synonymous with e-book.

iTex viewer[6] is a free application to use the digital contents which offer the medical study in iPad. To use this application, you can read the books of "medical study e-Text" in iPad.

Digital text[7], [8] is an educational application that the Ministry of Education of Japan considers to introduce to elementary schools and junior high schools by 2020. This application has basic functions of edit, movement, addition and elimination.

iOS[17] is a mobile operating system created and developed by Apple Inc. exclusively for its hardware. It is the operating system that presently powers many of the company's mobile devices, including the iPhone, iPad, and iPod touch. It is the second most popular mobile operating system globally after Android by sales.

We recall once again, the logicality is one of the most important studying mathematics of the school education. But these existing applications, specially, the digital text, are not logical. So we consider that the logical educational application is required.

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K. MORI, K. SHINANO, T. TSUCHIYA are School of Computer Science and Engineering The University of Aizu, Aizu-Wakamatsu 965-8580, JAPAN email: Kazuyoshi.MORI@IEEE.ORG.

II. Logical Education

There are five points that we consider to be important. These are "What we need to show", "Symmetric," "Detective," "Step by step" and "Interactive."

a) What we need to show: When we do not know procedure of proof, we cannot solve. Thus, we need to know what we need to show.

b) Symmetric: Description is sometimes in parallel. We can compare difference of proofs by making it symmetric visually.

c) Detective: Detective is to expect and argue unknown things with known things.

d) Step by step: We can obtain a solution to deduce gradually when we solve a problem.

e) Interactive: There are various methods to show solution of proof: For example, to show all of sentences, to show one line sentence at a time, to show sentences parallely, etc. So students can select a method as they can learn.

f) Hierarchical Approach: Our first point is "What we need to show," and then the next one is the deduction. However, to archive the deduction, we often have some the point we need to show. This is an hierarchical structure. We will consider the successive "What we need to show" and the deduction (See Figure 2).

III. Hierarchical Approach

We consider here an example of the hierarchical approach. Let us consider the set equation:

$$A \cup (B \cup C) = (A \cup B) \cup C, \tag{1}$$

where A, B, and C are sets. This is the associative low of the set operation.

Using Venn Diagram, (1) can be depicted as Figure 1. Th typical proof of (1) is to use Venn Diagram of Figure 1. Each set is given as an circle. The light areas in the left and right of the figure are to show $(B \cup C)$ of the left hand side and $(A \cup B)$ of the right hand side of (1), respectively. Then three-circled areas in the left and right of the figure are to show $A \cup (B \cup C)$ of the left hand side and $(A \cup B) \cup B$ of the right hand side of (1), respectively. They have same outlines, so that the sets $(A \cup B) \cup B$ and $A \cup (B \cup C)$ are equal to each other.

The proof by Venn Diagram is intuitive, which is much attractive. Even so, the proof by this style cannot be precise. It logically follows that *the proof by this style is not precise*.

Thus we need to give the descriptive proof, an example of which is shown in Figure 3.

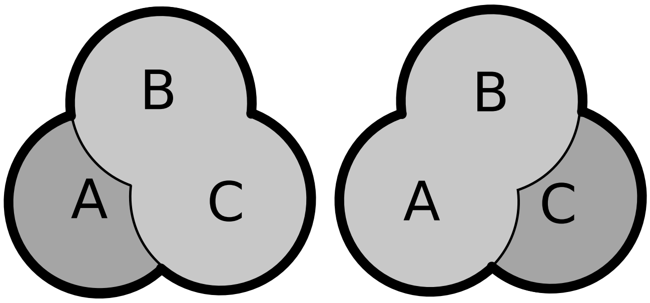

Fig. 1. Venn Diagram of $A \cup (B \cup C) = (A \cup B) \cup C$

Original "what we need to show" is

$$A \cup (B \cup C) = (A \cup B) \cup C. \quad (2)$$

In order to show it, we show

$$\text{(a) } A \cup (B \cup C) \subset (A \cup B) \cup C \text{ and,} \quad (3)$$

$$\text{(b) } A \cup (B \cup C) \supset (A \cup B) \cup C. \quad (4)$$

Now second "what we need to show"-s are (a) and (b). These (a) and (b) are to be "Symmetric."

Let us consider (a). For any set, the empty set is its subset. Thus, we can proceed to the case where $A \cup (B \cup C)$ is not empty. Let x be an element of $A \cup (B \cup C)$. This is split into two cases

$$\text{(a-1) } x \text{ is an element of } A \ (x \in A) \text{ and ,} \quad (5)$$

$$\text{(a-2) } x \text{ is an element of } B \cup C \ (x \in B \cup C). \quad (6)$$

These (a-1) and (a-2) are to be "Symmetric." Now the next "what we need to show" is to show

$$x \text{ is an element of } (A \cup B) \cup C \quad (7)$$

based on either (a-1) or (a-2).

In the case of (a-1), we have, by direct set deduction, $x \in A \cup B$ and one again by direct set deduction, $x \in (A \cup B) \cup C$.

In the case of (a-2), we have, further, another split, that is, (a-2-i) $x \in B$ or (a-2-ii) $x \in C$. These (a-2-i) and (a-2-ii) are "Symmetric."

In the case of (a-2-i), we have, by direct set deduction, $x \in A \cup B$ and one again by direct set deduction, $x \in (A \cup B) \cup C$.

In the case of (a-2-ii), we have, by direct set deduction, $x \in (A \cup B) \cup C$.

For all cases, we have $x \in (A \cup B) \cup C$, which implies $A \cup (B \cup C) \subset (A \cup B) \cup C$.

We have shown in Figure 4 the set equation (2) by the hierarchical approach.

IV. Implemented Application

We have implemented the proof of the associative law of sets, Equation 1, based on the hierarchical approach. The implemented system runs on Japanese as a local language because the system is offered to students of elementary school, junior high school, and high school.

A. *What we need to show*

We can know how to solve a problem by making clear what is goal of a problem (see Figure 5).

Also, we can see the local goal of the problem (see Figure 6). The red rectangles in the figure are local goals.

B. *Symmetric*

Figure 7 shows the whole proof of (1). This consists of two columns. One side shows "$A \cup (B \cup C) \supset (A \cup B) \cup C$." Other side shows "$A \cup (B \cup C) \supset (A \cup B) \cup C$." They are located symmetrically. As a result, learners can understand the parallelness of the description of the proof.

V. Result and Future Works

An educational application implemented in iOS based on the hierarchical approach. Implemented application includes "What we need to show," "Symmetric," "Detective," "Step by step" and "Interactive." One of future works is to add animation and venn diagram to implemented application. Another is to be desired that the educational application includes items, for example, contraposition, proof by contradiction and clarity of definition.

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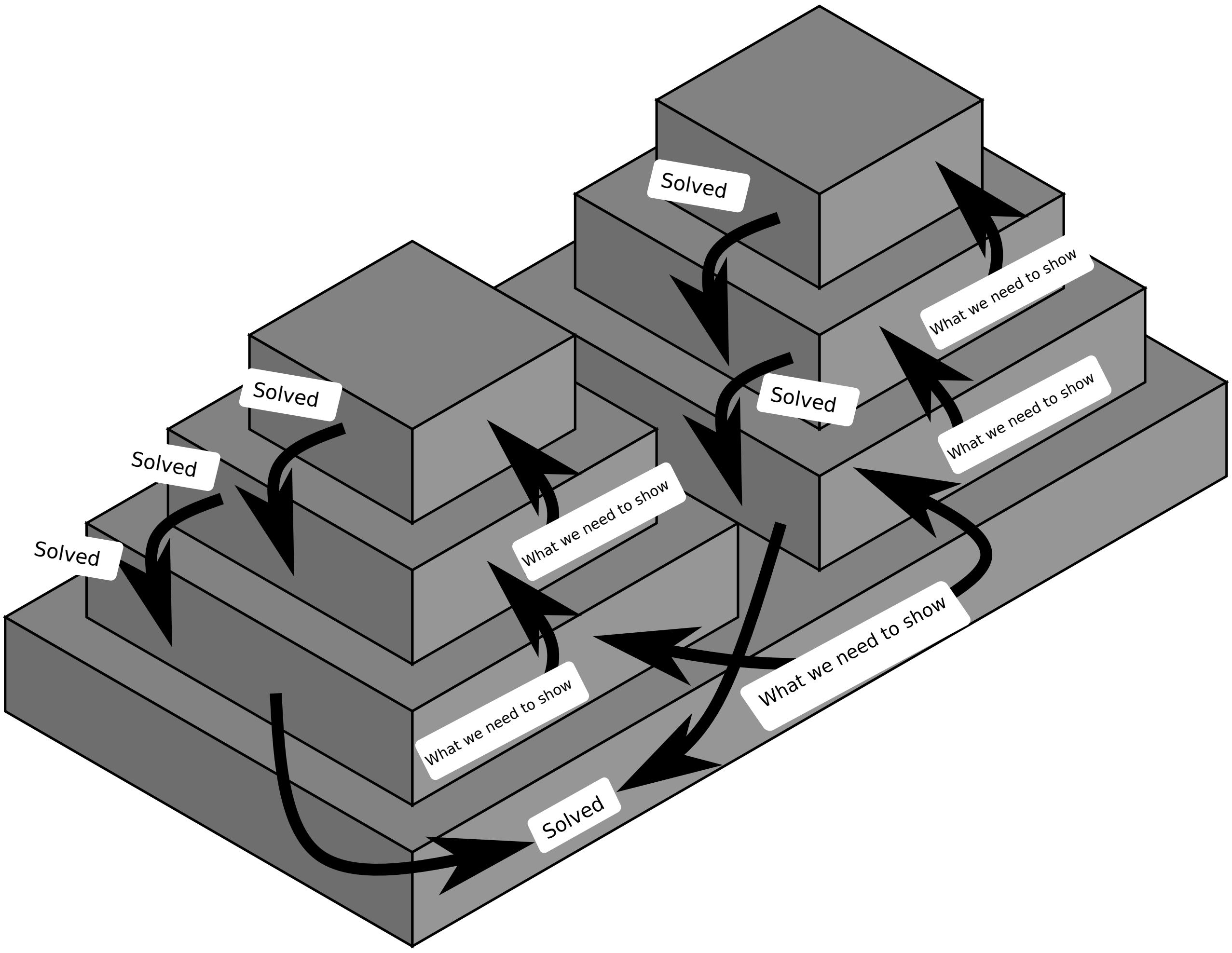

Fig. 2. Hierarchical Approach

We prove by showing both (a) $A \cup (B \cup C) \subset (A \cup B) \cup C$ and (b) $A \cup (B \cup C) \subset (A \cup B) \cup C$.

(a) If $A \cup (B \cup C)$ is the empty set, the inclusion is obvious. Thus, in the following, we suppose that $A \cup (B \cup C)$ is not empty.

Let x is an element of $A \cup (B \cup C)$ ($x \in A \cup (B \cup C)$). Then we have at least one of cases (a-1) x is an element of A ($x \in A$) and (a-2) x is an element of $B \cup C$ ($x \in B \cup C$).

(a-1) From $x \in A$, we have $x \in A \cup B$, and then $x \in (A \cup B) \cup C$.

(a-2) From $x \in B \cup C$, we have at least one of cases (a-2-i) x is an element of B ($x \in B$) and (a-2-ii) x is an element of C ($x \in C$).

(a-2-i) From $x \in B$, we have $x \in A \cup B$, and then $x \in (A \cup B) \cup C$.

(a-2-ii) From $x \in C$, we have $x \in (A \cup B) \cup C$.

For all cases, we have $x \in (A \cup B) \cup C$, so that $A \cup (B \cup C) \subset (A \cup B) \cup C$.

Analogously , we have $A \cup (B \cup C) \supset (A \cup B) \cup C$.

Now having (a) and (b), we have (1).

Fig. 3. Descriptive proof of $A \cup (B \cup C) = (A \cup B) \cup C$

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Show $A\cup(B\cup C)=(A\cup B)\cup C$.

- (a) Show $A\cup(B\cup C)\subset(A\cup B)\cup C$.
  - Suppose $A\cup(B\cup C)$ is empty.
    - Thus, $A\cup(B\cup C)\subset(A\cup B)\cup C$.
    - We have shown $A\cup(B\cup C)\subset(A\cup B)\cup C$ when $A\cup(B\cup C)$ is empty.
  - Suppose $A\cup(B\cup C)$ is not empty.
    - Let $x\in A\cup(B\cup C)$.
      - (a-1) Consider the case $x\in A$.
        - Then, $x\in A\cup B$.
          - Then, $x\in(A\cup B)\cup C$.
          - We have shown $x\in(A\cup B)\cup C$ when $x\in A$.
      - (a-2) Consider the case $x\in B\cup C$.
        - (a-2-i) Consider the case $x\in B$.
          - Then, $x\in A\cup B$.
            - Then, $x\in(A\cup B)\cup C$.
            - We have shown $x\in(A\cup B)\cup C$ when $x\in B$.
        - (a-2-ii) Consider the case $x\in C$.
          - Then, $x\in(A\cup B)\cup C$ directly.
          - We have shown $x\in(A\cup B)\cup C$ when $x\in C$.
        - We have shown $x\in(A\cup B)\cup C$ when $x\in B\cup C$.
      - We have shown $x\in(A\cup B)\cup C$ when $x\in A\cup(B\cup C)$.
    - We have shown $A\cup(B\cup C)\subset(A\cup B)\cup C$ when $A\cup(B\cup C)$ is not empty.
  - We have shown $A\cup(B\cup C)\subset(A\cup B)\cup C$ for all cases.
- (b) Show $A\cup(B\cup C)\supset(A\cup B)\cup C$.
  - Suppose $(A\cup B)\cup C$ is empty.
    - Thus, $A\cup(B\cup C)\supset(A\cup B)\cup C$.
    - We have shown $A\cup(B\cup C)\supset(A\cup B)\cup C$ when $(A\cup B)\cup C$ is empty.
  - Suppose $(A\cup B)\cup C$ is not empty.
    - Let $x\in(A\cup B)\cup C$.
      - Consider the case $x\in A\cup B$.
        - Consider the case $x\in A$.
          - Then, $x\in A\cup(B\cup C)$ directly.
          - We have shown $x\in A\cup(B\cup C)$ when $x\in A$.
        - (a-2-ii) Consider the case $x\in B$.
          - Then, $x\in B\cup C$.
            - Then, $x\in A\cup(B\cup C)$.
            - We have shown $x\in A\cup(B\cup C)$ when $x\in B$.
        - We have shown $x\in A\cup(B\cup C)$ when $x\in A\cup B$.
      - Consider the case $x\in C$.
        - Then, $x\in A\cup B$.
          - Then, $x\in A\cup(B\cup C)$.
          - We have shown $x\in A\cup(B\cup C)$ when $x\in C$.
      - We have shown $x\in A\cup(B\cup C)$ when $x\in(A\cup B)\cup C$.
    - We have shown $A\cup(B\cup C)\supset(A\cup B)\cup C$ when $(A\cup B)\cup C$ is not empty.
  - We have shown $A\cup(B\cup C)\supset(A\cup B)\cup C$ for all cases.

We have shown $A\cup(B\cup C)=(A\cup B)\cup C$.

Fig. 4. Set Equation $(A\cup B)\cup C=A\cup(B\cup C)$ by hierarchical approach

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Fig. 5. First “What we need to show” 2 (in Japanese)

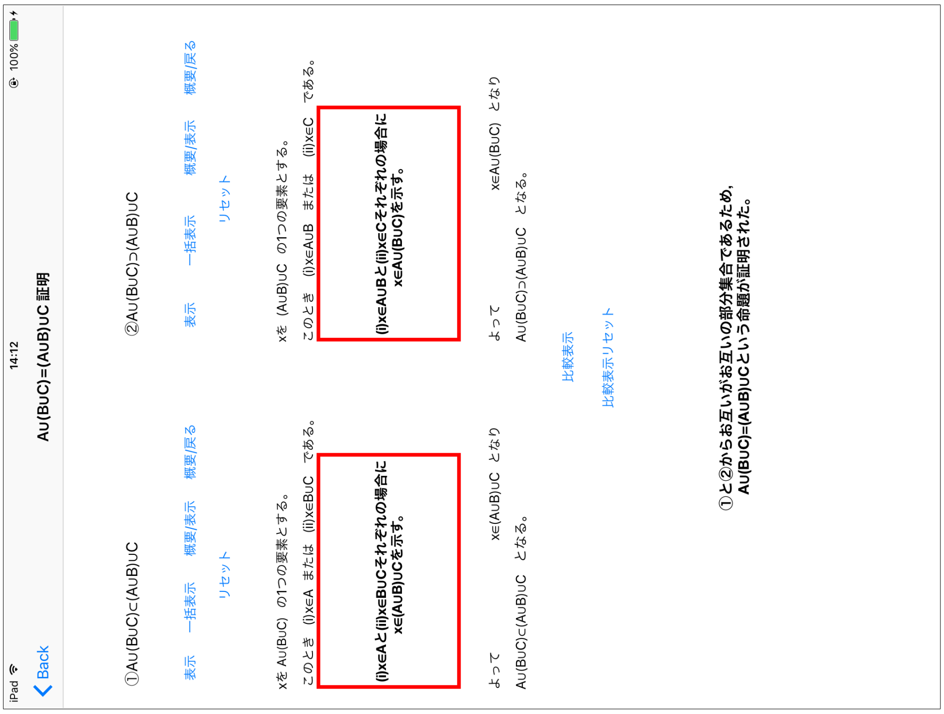

Fig. 6. First “What we need to show” 2 (in Japanese)

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Fig. 7. Symmetric structure of proof (in Japanese)

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